PHILOSOPHICAL

TRANSACTIONS,

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCXIV.

PART I.

LONDON,

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ADVERTISEMENT.

THE Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions*, take this opportunity to acquaint the Public, that it fully appears, as well from the council-books and journals of the Society, as from repeated declarations which have been made in several former *Transactions*, that the printing of them was always, from time to time, the single act of the respective Secretaries, till the Forty-seventh Volume: the Society, as a Body, never interesting themselves any further in their publication, than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the Public, that their usual meetings were then continued, for the improvement of knowledge, and benefit of mankind, the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed, to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions;* which was accordingly done upon the 26th of March, 1752. And the grounds of their choice are, and will continue to

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be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings, contained in the several papers so published, which must still rest on the credit or judgment of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body, upon any subject, either of Nature or Art, that comes before them. And therefore the thanks which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they receive them, are to be considered in no other light than as a matter of civility, in return for the respect shewn to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report, and even to certify in the public newspapers, that they have met with the highest applause and approbation. And therefore it is hoped, that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

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Meteorological Journal kept at the Apartments of the Royal Society.

The PRESIDENT and COUNCIL of the ROYAL SOCIETY adjudged the Medal on Sir GODFREY COPLEY's Donation, for the year 1813, to William Thomas Brande, Esq. F. R. S. for his Communications concerning the Alcohol contained in fermented Liquors, and other Papers printed in the Philosophical Transactions.

PHILOSOPHICAL TRANSACTIONS.

I. *A Synoptic Scale of Chemical Equivalents.* By William Hyde Wollaston, *M. D. Sec. R. S.*

Read November 4, 1813.

When the nature of any saline compound is proposed as the subject of inquiry to an analytic chemist, the questions that occur for his consideration are so varied and so numerous, that he will seldom be disposed to undertake a series of original experiments for the purpose of satisfying his inquiries, so long as he can rely upon the accuracy of those results that have been obtained by the labour of others, who have preceded him in this field of patient investigation.

If, for instance, the salt under examination be the common blue vitriol, or crystallized sulphate of copper, the first obvious questions are, (1) How much sulphuric acid does it contain? (2) How much oxide of copper? (3) How much water? He may not be satisfied with these first steps in the analysis, but may desire to know further the quantities (4) of sulphur, (5) of copper, (6) of oxygen, (7) of hydrogen. As means of gaining this information, he naturally considers the quan-

tities of various reagents that may be employed for discovering the quantity of sulphuric acid, (8) how much barytes, (9) carbonate of barytes, or (10) nitrate of barytes, would be requisite for this purpose; (11) How much lead is to be used in the form of (12) nitrate of lead; and when the precipitate of (13) sulphate of barytes or (14) sulphate of lead are obtained, it will be necessary that he should also know the proportion which either of them contains of dry sulphuric acid. He may also endeavour to ascertain the same point by means of (15) the quantity of pure potash, or (16) of carbonate of potash requisite for the precipitation of the copper. He might also use (17) zinc or (18) iron for the same purpose, and he may wish to know the quantities of (19) sulphate of zinc, or (20) sulphate of iron that will then remain in the solution.

These, and very many more questions of the same kind, which it would be tedious to specify, and needless to enumerate, engage the thoughts, and will occupy much of the time of every experimental chemist, unless he can have recourse to some record of former analyses on which he can depend.

The scale, which I am about to describe, is designed to answer at one view all these questions, with reference to most of the salts contained in the table, not merely expressing numerically the proportions by which the desired answers may be calculated, but directly indicating the actual weights of the several ingredients, contained in any assumed weight of the salt under consideration, and also the actual quantities of several reagents that may be used, and of the precipitates that would be obtained by each.

In the formation of this scale, it is requisite in the first place

to determine the proportions in which the different known chemical bodies unite with each other, and to express these proportions in such terms that the same substance shall always be represented by the same number.

It is to RICHTER that we are originally indebted for this mode of expression, and for having first observed that law of permanent proportions on which the possibility of this numerical representation is founded. The proportions assigned to various salts by his predecessors BERGMAN, WENZEL, KIRWAN, were incompatible with this mode of notation. If we turn to BERGMAN's treatise *De Analysi Aquarum*, we find it stated that in sulphate of potash 40 of acid are combined with 52 of potash, or that 100 of sulphuric acid take 130 of potash. In muriate of potash, 61 of the alkali are said to be combined with 31 of acid, which is in the proportion of 130 to 66. So that the same quantity of potash that is saturated by 100 sulphuric acid, requires of muriatic 66.

But if we make a similar estimate by means of lime, since sulphate of lime is said to contain 46 acid combined with 32 lime, 100 of acid would require 69,5. And in muriate of lime, since 44 of lime are said to be combined with 31 of acid, thence 69,5 of lime would require 49. So that in this instance it would appear that the equivalent to 100 sulphuric acid, instead of being 66 muriatic, is 49; which, if true, would defeat our attempts to express the same body always by the same number.

In comparing the analyses of WENZEL with each other, we find the same inconsistency. If we select sulphate of ammonia, and muriate of ammonia, we obtain 67,3 as the equivalent of muriatic acid. But by comparison of sulphate of magnesia

with muriate of magnesia, it would appear to be 73 instead of 67,3.

In recurring to the tables of KIRWAN, a similar obstacle presents itself to the determination of the quantity of muriatic acid, that is equivalent to a given weight of sulphuric acid. When the comparison is made by means of potash, the result would make it appear that 68,8 is the relative weight of muriatic acid. But, if the compounds of these acids with lime be employed in the computation, the result instead of 68,8 gives only 59.

RICHTER remarked, on observing this sort of inconsistency, that if sulphate of potash formed according to the proportions of KIRWAN were decomposed by muriate of lime, there should be found a large excess of alkali in the solution. But, on the contrary, by direct experiment he found that neutral salts, when mixed, remained in all cases neutral, and consequently, that the same weight of muriatic acid would in all cases be found equivalent to the same quantity of sulphuric acid; and therefore might be conveniently expressed in stating the composition of salts, by the same number. He estimates this acid at 712, as the equivalent to 1000 dry sulphuric acid, the number assumed as his standard of comparison, to which all other numbers for acids, alkalies, and earths are adapted.

It could not escape the penetration of M. BERTHOLLET, that there exist numerous deviations from this law of neutralization, and cases of prevailing affinity dependent on a redundance of one or other ingredient in a mixture of salts. But he was not so happy in detecting the definite law, by which many, at least, of these deviations are governed. It has since been found, that when a base unites with a larger portion of acid

than is sufficient to saturate it, the quantity combined is then an exact simple multiple of the former, thus exhibiting a new modification of the law of definite proportions, rather than any exception to it.

The first instance in which the same body was supposed to unite with different doses of another, in such proportions that one of these doses is a simple multiple of the other, was noticed by Mr. HIGGINS, who conceived, rather than actually observed to occur, certain successive degrees of oxidation of azote, and represented the series of its combinations with oxygen to be

* Azote 1 with 2 oxygen making nitrous gas.

Azote 1 with 3 oxygen making red nitrous vapour.

Azote 1 with 4 oxygen making yellow nitrous acid.

Azote 1 with 5 oxygen making white nitric acid.

He at the same time added his opinion, that such are the proportions in which these gases unite to each other by *bulk*, having before observed one instance of union by exactly double bulk in the formation of water by the combustion of hydrogen and oxygen, and expressed his persuasion that the number of particles in a given bulk of the different gases is the same, and that the number of particles in the compounds of azote and oxygen, are successively in the proportions above stated.

But though Mr. HIGGINS, in the instance of the union of hydrogen with oxygen, anticipated the law of bulks observed by M. GAY LUSSAC, with respect to the union of gases, and in his conception of union by ultimate particles clearly preceded Mr. DALTON in his atomic views of chemical combination, he appears not † to have taken much pains to ascertain

* A comparative View of the phlogistic and antiphlogistic theories, 1789, p. 133.

† In straw-coloured nitrous acid, the proportion *appears* to be four to one; but

the actual prevalence of that law of multiple proportions by which the atomic theory is best supported, and it is in fact to Mr. DALTON that we are indebted for the first correct observation of such an instance of a simple multiple in the union of nitrous gas with oxygen. In his endeavours to determine the composition of the atmosphere, he found that the quantity of oxygen contained in 100 measures of common air would combine with either 36 or 72 measures of nitrous gas, according to certain variations in the mode of conducting the experiment.*

Chemists in general, however, appear to have been by no means duly impressed with the importance of this observation of Mr. DALTON, till they were in possession of other facts observed by Dr. THOMSON and myself,† in a more tangible form, with regard to neutral and superacid or subacid salts, which could be made the subjects of more deliberate and less equivocal experiments; and it is, perhaps, owing to the repetition and confirmation of them by M. BERTHOLLET,‡ that they have attracted the attention of other chemists, who are now ready to admit that the term binacid correctly expresses the relation of many superacid salts to neutrals consisting of the same ingredients. Since that time the additional instances in which the same law has been observed to prevail, are become so numerous, especially with regard to different degrees of oxidation, that we have the greatest reason to presume that

the colourless contains *about* five of dephlogisticated to one of phlogisticated air. Comparative View, p. 84.

* Manchester Mem. Vol. V.—NICH. Journal, Vol. XIII. p. 433.

† Phil. Trans. 1808. p. 74.—Ditto, p. 96.

‡ Mem. d'Arcueil. Tome II. p. 470.

it is universal, and that in such analyses as are found not to accord with this general observation, we are warranted in suspecting some degree of inaccuracy in one or other of the results compared together.

According to Mr. DALTON's theory, by which these facts are best explained, chemical union in the state of neutralization takes place between single atoms of the substances combined; and in cases where there is a redundance of either ingredient, then two or more atoms of this kind are united to only one of the other.

According to this view, when we estimate the relative weights of equivalents, Mr. DALTON conceives that we are estimating the aggregate weights of a given number of atoms, and consequently the proportion which the ultimate single atoms bear to each other. But since it is impossible in several instances, where only two combinations of the same ingredients are known, to discover which of the compounds is to be regarded as consisting of a pair of single atoms, and since the decision of these questions is purely theoretical, and by no means necessary to the formation of a table adapted to most practical purposes, I have not been desirous of warping my numbers according to an atomic theory, but have endeavoured to make practical convenience my sole guide, and have considered the doctrine of simple multiples, on which that of atoms is founded, merely as a valuable assistant in determining, by simple division, the amount of those quantities that are liable to such definite deviations from the original law of RICHTER.

Having some time since computed for private use a series of supposed atoms, I had assumed oxygen as the decimal unit of my scale, in order to facilitate the estimation of those

numerous combinations which it forms with other bodies. But, though in the present table of Equivalents, I have retained the same unit, and have taken care to make oxygen equally prominent for the same reason as before, as well as on account of the important part it performs in determining the affinities of bodies by the different proportions in which it is united to them; nevertheless the real measure, by which most bodies are compared to each other, in any experiments that I have made, and to which I have, in fact, endeavoured to find equivalents, is a determinate quantity of carbonate of lime. This is a compound, that may be regarded as most distinctly neutral. It is most easy to obtain in a state of uniform purity; most easy to analyse (as a binary compound); it is a most convenient measure for the powers of acids, and affords the most distinct expression for the comparative neutralizing powers of alkalies.

The first question, consequently, to be resolved is, by what number are we to express the relative weight of carbonic acid, if oxygen be fixed at 10. It seems to be very well ascertained, that a given quantity of oxygen yields exactly an equal measure of carbonic acid by union with carbon; and since the specific gravities of these gases are as 10 to 13,77,* or as 20 to 27,54, the weight of carbon may be justly represented by 7,54, which, in this instance, is combined with 2 of oxygen forming the deutoxide, and carbonic oxide being the protoxide will be duly represented by 17,54.

Carbonic acid having consequently been assumed as 27,54, it follows from the analysis of carbonate of lime, which by heat loses 43,7 per cent. of acid, and leaves 56,3 of base, that they

* BIOT and ARAGO 1,1036 : 1,1956 :: 10 : 13,77.

are combined in the proportion of 27,54 to 35,46, and consequently that lime must be represented by 35,46, and carbonate of lime by 63.

If I would proceed in the series for the purpose of estimating the reliance to be placed on preceding analyses, I might dissolve 63 of carbonate of lime in muriatic acid, and by evaporating to perfect dryness should obtain about 69,56* muriate of lime, and by deducting the weight of the lime 35,46, should learn, by means of the difference 34,1, what is to be considered as dry muriatic acid.

But since lime is now known, by the brilliant discoveries of Sir H. Davy, to be a metallic body united with oxygen, this salt may also be viewed as a binary compound in a different light as oxymuriate of calcium; in which case we must transfer the weight of 10 oxygen to the muriatic acid, making 44,1 of oxymuriatic acid combined with 25,46 calcium. Or, lastly, if with the same distinguished chemist, we regard it as chlorid of calcium, its place in the scale of equivalents is the same 69,56, and the portion of matter here added to the calcium, whether it retain its late name of oxymuriatic acid, or revert to its original one of dephlogisticated marine acid, or assume its new one of chlorine, will be rightly represented by 44,1, which expresses a bare fact without reference to any theory, and affords the means of estimating the proportion of this constituent in all muriatic compounds, without need of controversy respecting its simple or compound nature, which may

* In Dr. Marcet's experiments on the composition of muriate of lime, referred to in his Analysis of the Water of the Dead Sea,

50,77 carbonate gave 56,1 of muriate of lime,

and 50,77 : 56,1 :: 63 : 69,6.

never admit of any argument that will be deemed conclusive by all parties.

With the same latitude of interpretation may be understood muriate of potash or of soda in the scale of equivalents; and the relative weights of mere potash or soda may, perhaps, be determined better by means of these compounds than by any other, because they are not liable to be superacid, and are not decomposed by heat.

If to a quantity of muriatic acid, which, by previous trial, I know would dissolve 100 carbonate of lime, I add 100 grains of crystallized carbonate of potash, and after the addition find that it will dissolve only 49,8 of carbonate of lime; I hence infer that 100 of this carbonate is equivalent to 30,9 carbonate of lime, and consequently that 125.5 is the equivalent to 63 in the table.

Next, if I combine 125.5 of crystallized carbonate of potash with an excess of muriatic acid, and evaporate to dryness, I expel the whole of the water with all redundant acid, and I find 93,2 of neutral salt; and whether I call it muriate of potash, or chlorid of potassium, or by any other name, with any other views, I may deduct 34,1 as dry muriatic acid, (whether real or imaginary*) and infer the equivalent for potash to be 59,1, even though there should, in fact, be only 49,1 of potassium present, requiring 10† of oxygen to convert it into potash.

* Its separate existence is certainly imaginary, for it can no more be obtained uncombined than dry sulphuric acid, or dry nitric acid.

† If all the steps in the series, by which the number 49,1 is inferred, be correct, this should be exactly 10,00 without any fraction; and the proportion assigned to muriate of potash by BERZELIUS is sufficiently near, to shew that there can be no considerable error. 83,02 : 16,58 :: 49,1 : 10,04.

The next question that occurs relates to the composition of this crystallized carbonate of potash, which I am induced to call bi-carbonate of potash, for the purpose of marking more decidedly the distinction between this salt and that which is commonly called a subcarbonate,* and in order to refer at once to the double dose of carbonic acid contained in it. With reference to carbonate of lime also, I must necessarily consider it as a supercarbonate, for if I add a solution of this salt to a neutral solution of muriate of lime, a considerable effervescence takes place, from a redundance of carbonic acid beyond what is necessary to saturate the lime. If I saturate 125,5 of this salt with nitric acid, taking due precautions not to expel any of the fluid along with the gas which escapes, it loses about 55 of carbonic acid, which is the double of 27,5. But if, previous to the saturation, I heat the salt moderately red, it loses 38,8, consisting of 27,5 carbonic acid and 11,3 water, after which the addition of an acid expels only 27,5, or a single dose of carbonic acid.†

I have in this experiment made use of nitric acid, in order that the resulting compound might guide me in the selection from among former estimates which are extremely discordant with regard to the equivalent of that acid. The proportion of nitrate of potash, which I have obtained by evaporating such a solution by a heat just sufficient to fuse the residuum, gave at the lowest in three experiments 126, for the equivalent of nitrate of potash; from which, if we deduct 59,1 potash,

* I avoid using the term carbonate of potash for either of these salts, because it has been applied to both, and consequently is liable to be misunderstood when standing alone.

† Phil. Trans. 1808, p. 97.

there will remain 66,9 as the apparent equivalent of dry nitric acid. Consequently, I have no hesitation in preferring the estimate * to be obtained from RICHTER's analysis of nitrate of potash, which gives 67,45, from which if we subtract one portion of azote 17,54, there remain 49,91, so nearly 5 portions of oxygen, that I consider the truth to be 17,54 + 50, or 67,54.

From this sketch of the mode in which such an inquiry may be pursued, wherever it is necessary to make any original experiments, it will be fully understood what is meant by equivalents, and in what manner the series might be continued. I have, however, in most instances drawn my inferences from former analyses, and indeed in all, where I could find coincidences between different authorities sufficient to give confidence in their results.

But with respect to oxalic acid, I again found a difficulty in deciding among the discordant results of different analyses, and was obliged to have recourse to direct experiment.

100 grains of bin-oxalate of potash (commonly called salt of sorrel) were subjected to a degree of heat sufficient to destroy the oxalic acid, and to convert the salt into a subcarbonate of potash. A quantity of muriatic acid was then poured on this residuum, and afterwards saturated with carbonate of lime; and an equal quantity of the same acid was saturated with carbonate of lime alone. By the excess of carbonate dissolved in the latter instance, it was found that 100 bin-oxalate was equivalent to 40,9 carbonate of lime; and hence the equivalent to 63 carbonate of lime will be 154 of the bin-oxalate of potash. After deducting 59,1 potash, the remainder

* 46,7 : 53,3 :: 59,1 : 67,45 quoted in Mem. d'Arcueil, II. 59.

94,9 divided by 2, gives 47,45 for the equivalent of dry oxalic acid. I therefore again adopt the result of the very industrious and ever accurate BERZELIUS, obtained by means of oxalate of lead, that 296,6* litharge are combined with 100 oxalic acid, which are in the proportion of 139,5 litharge to 47,0 oxalic acid. Such a degree of accordance between methods totally different appears highly satisfactory, and seems to shew that in attempts to determine the same point by means of lime, some compounds may possibly be formed at the same time differing in the proportions of acid and base, as in the cases of oxalate and bin-oxalate of strontia, observed by Dr. THOMSON, and that erroneous inferences may have been drawn from precipitates in which they are blended.

With the exception of those instances that I have enumerated, there are few in which I have found it necessary to make any new experiments, as I have met with coincidences between the independent results of others sufficient to satisfy me of their correctness; and accordingly I have adopted such determinations without any pretensions to improve upon them by new experiments of my own.

It is not my design in the table which follows this paper, to attempt a complete enumeration of all those elements or compounds which I suppose to be well ascertained, but merely to include some of those which most frequently occur. I do not offer it as an attempt to correct the estimates that have been formed by others, but as a method in which their results may be advantageously applied in forming an easy approximation to any object of our inquiries.

The means by which this is effected may be in part understood by inspection of the Plate I., in which will be seen the

* Ann. de Chimie, No. 243.

list of substances intended to be estimated, arranged on one or the other side of a scale of numbers in the order of their relative weights, and at such distances from each other, according to their weights, that the series of numbers placed on a sliding scale can at pleasure be moved, so that any number expressing the weight of a compound may be brought to correspond with the place of that compound in the adjacent column. The arrangement is then such, that the weight of any ingredient in its composition, of any reagent to be employed, or precipitate that might be obtained in its analysis, will be found opposite to the point at which its respective name is placed.

In order to shew more clearly the use of this scale, the Plate exhibits two different situations of the slider, in one of which oxygen is 10, and other bodies are in their due proportion to it, so that carbonic acid being 27,54, and lime 35,46, carbonate of lime is placed at 63.

In the second figure, the slider is represented drawn upwards till 100 corresponds to muriate of soda; and accordingly the scale then shews how much of each substance contained in the table is equivalent to 100 of common salt. It shews, with regard to the different views of the analysis of this salt, that it contains 46,6 dry muriatic acid, and 53,4 of soda, or 39.8 sodium, and 13,6 oxygen; or if viewed as chlorid of sodium, that it contains 60,2 chlorine, and 39,8 sodium. With respect to reagents, it may be seen that 283 nitrate of lead, containing 191 of litharge employed to separate the muriatic acid, would yield a precipitate of 237 muriate of lead, and that there would then remain in solution nearly 146 nitrate of soda. It may at the same time be seen, that the acid in this quantity of salt would serve to make 232 corrosive sublimate contain-

ing 185,5 red oxide of mercury, or would make 91,5 muriate of ammonia, composed of 6 muriatic gas (or hydromuriatic acid) and 29,5 ammonia. The scale shews also, that for the purpose of obtaining the whole of the acid in distillation the quantity of oil of vitriol required is nearly 84, and that the residuum of this distillation would be 122 dry sulphate of soda, from which might be obtained, by crystallization, 277 of GLAUBER salt containing 155 water of crystallization. These and many more such answers appear at once by bare inspection, as soon as the weight of any substance intended for examination is made by motion of the slider correctly to correspond with its place in the adjacent column.

With respect to the method of laying down the divisions of this scale, those who are accustomed to the use of other sliding-rules, and are practically acquainted with their properties, will recognise upon the slider itself the common GUNTER's line of numbers, (as it is termed) and will be satisfied that the results which it gives are the same that would be obtained by arithmetical computation.

Those who are acquainted with the doctrine of ratios, and with the use of logarithms as measures of ratios, will understand the principle on which this scale is founded, and will not need to be told that all the divisions are logometric, and consequently that the mechanical addition and subtraction of ratios here performed by juxta-position, corresponds in effect to the multiplication and division of the numbers by which those ratios are expressed in common arithmetical notation.

To others who are not equally conversant with the nature of logarithms, and consequently have not so correct a conception of the magnitudes of ratios, some further explanation of

the mode in which the scale of equivalents is constructed, will, I presume, be acceptable.

They will observe, that the series of natural numbers are not placed at equal intervals on the scale; but that at all equal intervals are found numbers which bear the same proportion to each other. In fig. 3, some of the larger intervals alone are represented on a line similarly divided. The succession of intervals, marked A, B, C, D, E, are all equal, and at these points of division are placed numbers 1, 2, 4, 8, 16, which increase progressively by the same ratio. And since the series 3 : 6 : 12 : 24 increase in the same ratio of 1 to 2, these intervals *a*, *b*, *c*, *d*, *e*, are the same as the former. At another succession of different yet equal intervals, marked F, G, H, I, are placed numbers 1, 3, 9, 27, which increase regularly by an equal ratio of 1 to 3; and by means of a pair of compasses it would be found that the interval from 2 to 6, or from 6 to 18 (which are in the same ratio of 1 to 3,) is exactly equal to FG, the interval between 1 and 3. As any single space represents any one ratio, so the sum of any 2 or 3 equal spaces represent a double or triple ratio. If 1 be increased three times by the ratio of 1 to 2, it becomes 8, which bears to 1 triple the ratio of 2 to 1. This ratio is therefore rightly represented by AD, which is the triple of AB.

The distances of the intermediate numbers 5, 7, 10, 11, 13, &c. from 1 are likewise made proportional to the ratios which they bear to 1, and are easily laid down by means of a table of logarithms; for as these are arithmetic measures of the ratios which all numbers bear to unity, the spaces proportional to them become linear representations of the same quantities.

As the entire spaces AD, AE represent the ratios of 8 and of 16 respectively to 1, so the difference DE represents the ratio of 8 and 16, which stand at D and E, to each other. And in the same manner, any other space *kl* represents correctly the ratio of 7 to 13; so that the measure of a fraction expressed by quantities that are incommensurate is rendered as obvious to sight, as that of any simple multiple. And if a pair of compasses be opened to this interval, and transferred to any other part of the scale, the points of the compasses will be found to rest upon numbers bearing the same proportion to each other as those from which the interval was transferred.

It is exactly in this manner, that the various points in the column of equivalents indicate the several quantities sought in any given position of the slider. The relative distances, at which the articles are placed, represent so many different openings of the compasses rendered permanent and presented to view at once. In the table, which I shall place at the end of this communication, the relation of the various substances enumerated to each other is expressed by numbers. In the engraved scale of equivalents, the ratios of these numbers are represented by logometric intervals at which they are placed, their several positions being determined by those of their respective numbers on the slider, which is logometrically divided. Consequently all the several points in the column of equivalents will indicate numbers in the same due proportions to each other, whatever part of the scale may be presented to them. Those who seek information may obtain it by inspection; those who already possess it, may be able to correct the positions of some articles by direct comparison with the best

analyses upon record, in whatever numbers the results of those analyses may happen to be expressed.

I hope, that without trespassing too much on the time of the Society, I shall have rendered the principle and practical use of this scale intelligible. I trust, that it will prove useful as an assistant to chemists in general. It will at least serve for a specimen of the extreme facility of mechanical approximation, which may very frequently be advantageously substituted for computations, that are often more laborious than the accuracy of our data warrants; and if it tend to introduce into more general use, that valuable instrument the common sliding-rule, it will be the means of saving no inconsiderable portion of time to those who are engaged in scientific pursuits.

Numerical Table of Equivalents.

Hydrogen	(*a*)	1,32					
Oxygen		10,00					
Water		11,32					
Carbon	(*b*)	7,54	+ 10	Oxygen	= 27,54	Carbonic acid.	
Sulphur	(*f*)	20,00	+ 30	Oxyg.	= 50	Sulphuric acid.	
Phosphorus	(*g*)	17,40	+ 20	Oxyg.	= 37,4	Phosphoric acid.	
Azote	(*e*)	17,54	+ 50	Oxyg.	= 67,54	Nitric acid (*q*).	
Muriatic Acid (dry)	(*e*)	34,1	+ 10	Oxyg.	= 44,1	Oxymuriatic acid.	
Chlorine		44,1	= Oxymur. acid	+ 1,32	Hydrog.	= 45,42	Muriatic gas.
Oxalic acid	(*b*)	47,0					
Ammonia	(*p*)	21,5					
Soda	(*l*)	39,1	— 10	Oxyg.	= 29,1	Sodium.	
Potash	(*m*)	59,1	— 10	Oxyg.	= 49,1	Potassium.	
Magnesia	(*n*)	24,6					
Lime	(*c*)	35,46	+ 10	Oxyg.	= 25,46	Calcium.	
Strontia	(*k*)	69,					
Barytes	(*i*)	97,					
Iron	(*r*)	34,5	+ 10	Oxyg.	= 44,5	Green Oxid of Iron.	
			+ 15	Oxyg.	= 49,5	Red Oxid.	

Copper -	(*l*)	40.	+ 10	Oxyg.	= 50,	Black Oxid of Copper.
Zinc - -	(*s*)	41	+ 10	Oxyg.	= 51,	Oxid of Zinc.
Mercury -	(*v*)	125,5	+ 10	Oxyg.	= 135,5	Red Oxid Mercury.
					+ 125,5 ☿ = 261	Protoxid ☿.
Lead - -	(*d*)	129,5	+ 10	Oxyg.	= 139,5	Litharge.
Silver -	(*u*)	135,	+ 10	Oxyg.	= 145,	Oxid Silver in Muriate.
Subcarb. of Ammonia		49,0	+ 27,5	C. acid	= 76,5	Bi-Carb. of Ammonia.
Subcarb. of Soda -		66,6	+ 27,5	C. acid + 11,3	Water = 105,5	Bi-Carb. of Soda.
Subcarb. of Potash -		86,	+ 27,5	C. acid + 11,3	Water = 125,5	Bi-Carb. of Potash.
Carbonate of Lime		63				
——— Barytes -		124,5				
——— Lead -		167,				
Sulphuric acid (dry)		50.	+ 1 Water	11,3 = 61,3	Oil of Vitriol (sp. gr. 1,85).	
Sulphate of Soda -		89,1	+ 10 Water	113,2 = 202,3	Glauber Salt.	
——— Potash -		109,1				
——— Magnesia (*n*)		74,6	+ 7 Water	79,3 = 153,9	Epsom Salt.	
——— Lime		85,5	+ 2 Water	22,64 = 108,1	Selenite.	
——— Strontis -		119,0				
——— Barytes -		147,0				
——— Copper -		156,6	= 1 Acid + 1 Oxid.	+ (*v*) 5 Water	56,6.	
——— Iron -		173,8	= 1 Acid + 1 Oxid.	+ 7 Water	79,3.	
——— Zinc -		180,2	= 1 Acid + 1 Oxid.	+ 7 Water	79,3.	
——— Lead -		189,5				
Nitric acid (dry)	(*q*)	67,54	+ 2 Water	22,64 = 90,2	Liquid Nitric acid (sp. gr. 1,50).	
Nitrate of Soda -		106,6				
——— Potash -		126,6				
——— Lime -		103,0				
——— Barytes -		164,5				
——— Lead -		207,0				
Muriate of Ammonia -		66,9	= 1 Acid + 1 Amm. + 1 Water.			
——— Soda -		73,2				
——— Potash -		93,2	+ 60 Oxygen = 153,2	Hyper-Oxymuriate of Potash.		
——— Lime -		69,6				
——— Barytes -		131,0	+ 2 Water 22,6 (*n*) 153,6	Crystallized Mur. Barytes.		
——— Lead -		173,6				
——— Silver -		179,1				
Corrosive Sublimate -		170,1	= 1 Acid 1 Oxyg. + 1 Mercury.			
Calomel - -		296,1	= 1 Acid + 1 Oxyg. + 2 Mercury.			
Phosphate of Lead -		176,9				
Oxalate of Lead -		186,5				
Bin-Oxalate of Potash		153,0	= 2 Acid + 1 Potash			

Data on which the Table is founded.

								10,00		Oxygen
Composition of Water	(1)	88,286	:	11,714	:: Oxygen	10	:	1,327		Hydrogen (*a*)
								11,327		Water
Specific Gravities -	(2)	1,1036	:	1,5196	:: 2 Oxygen	20	:	27,54		Carbonic acid (*b*)
Carbonate of Lime	(3)	43,7	:	56,3	:: Carb. acid	27,54	:	35,46		Lime (*c*)
Carbonate of Lead	(4)	16,5	:	83,5	:: Carb. acid	27,54	:	139,5		Litharge
								10		
								129,5		Lead (*d*)
Litharge - -	(5)	7,15	:	92,85	:: Oxygen	10	:	129,7		
Muriate of Lime from Carbonate of Lime	(6)	50,77	:	56,1	:: Carb. Lime	63	:	69,6		Mur. Lime
						Lime		35,5		
								34,1		Muriatic acid (*e*)
Muriate of Lead -	(7)	409,47	:	100	:: Litharge	139,5	:	34,1		
Sulphate of Lead -	(8)	279	:	100	:: Litharge	139,5	:	50,0		Sulphuric acid
						3 Oxygen =		30		
Galena - -	(9)	86,64	:	13,36	:: Lead	129,5	:	20		Sulphur (*f*)
Ditto -	(10)	85,1	:	13	:: Lead	129,5	:	19,8		
Phosphate of Lead	(11)	380,56	:	100	:: Litharge	139,5	:	37,4		Phosphoric acid
						2 Oxygen		20		
Phosphoric acid -	(12)	53,28	:	46,72	:: Phosph. acid	37,4	:	17,4		Phosphorus (*g*)
Oxalate of Lead -	(13)	296,6	:	100	:: Litharge	139,5	:	47,0		Oxalic acid (*b*)
Carbonate of Barytes	(14)	100	:	352,57	:: Carb. acid	27,54	:	97		Barytes (*i*)
Sulphate of Barytes	(15)	34	:	66	:: Sulph. acid	50	:	97		
Sulphate of Strontia	(16)	42	:	58	:: Sulph. acid	50	:	69		Strontia (*k*)
Common Salt -	(17)	134	:	88	:: Chlorine	44,1	:	29		Sodium (*l*)
						10		10		
Ditto - -	(18)	100	:	114,78	:: Mur. acid	34,1	:	39,1		Soda
Subcarbonate of Soda	(19)	41,24	:	58,76	:: Carb. acid	27,54	:	39,1		
Muriate of Potash	(20)	100	:	173,47	:: Mur. acid	34,1	:	59,1		Potash (*m*)
								10		
								49,1		Potassium
					add Oxymuriatic acid or Chlorine			44,1		
Muriate of Potash from Potassium	(21)	32	:	60,8	:: Potassium	49,0	:	93,2		Mur. Potash

(1) Biot and Arago (2) Ditto (3) Experiment (4) Berzelius (5) Berzelius
(6) Marcet (7) Berzelius (8) Berzelius (9) Ditto (10) Thomson
(11) Berzelius (12) Rose (13) Berzelius (14) Ditto (15) Klaproth
(16) Ditto (17) Davy (18) Berzelius (19) Ditto (20) and (21) Ditto

Sulphate of Magnesia	(22)	67	: 33	::Sulph. acid	50	: 24,6	Magnesia (n)		
						50			
						74,6	} Sulph. Magnesia		
Epsom Salt 100 -	(23)	51,5	: 48,5	::7 Water	79,3	: 74,4			
Specific Gravities -	(24)	,07321	: 96913	::Hydrogen	1,327	: 17,54	Azote (o)		
				3 Hydrogen = 3 × 1,327		3,98			
Ammonia -	(25)	1 Azote + 3 Hydrogen				= 21,52	} Ammonia (p)		
Subcarb. Ammonia	(26)	56,02	: 43,98	::Carb. acid	27,54	: 21,6			
Bi-carb. Ammonia	(27)	11,8	: 28,2	::2 Carb. acid	55,1	: 21,6			
Nitrate of Potash -	(28)	46,7	: 53,3	::Potash	59,08	: 67,45	} Nitric acid (q)		
Nitric acid -	(29)	1 Azote + 5 Oxygen	17,54+50			= 67,54			
			2 Water = 2 × 11,32			= 22,64			
						90,18	} { Liquid Nitric acid		
Marble dissolved -	(30)	476	: 681,4	::Carb. Lime	63	: 90,23	{ S. G. 1,50		
Oxid of Iron -	(31)	22,5	: 77,5	::Oxygen	10	: 34,5	} Iron (r)		
						10			
Sulphate of Iron -	(32)	28,9	: 25,7	::Sulph. acid	50	: 44,5	} Oxid of Iron		
Oxid of Zinc -	(33)	24,41	: 100	::Oxygen	10	: 41	Zinc (s)		
Black Oxid Copper	(34)	20	: 100	::Oxygen	10	: 40	} Copper (t)		
						10			
Sulphate of Copper	(35)	32	: 32	::Sulph. acid	50	: 50	} Oxid of Copper		
Muriate Silver -	(36)	19,05	: 80,95	::Mur. acid	34,1	: 145	Oxid of Silver		
						10			
						135	} Silver (u)		
Horn Silver -	(37)	24,5	: 75,5	::Chlorine	44,1	: 136			
Sulphuret Silver -	(38)	14,7	: 100	::Sulphur	20	: 136			
Red Oxid of Mercury	(39)	8	: 100	::Oxygen	10	: 125	} Mercury (v)		
Ditto - -	(40)	30	: 380	::Oxygen	10	: 126,6			
Corros. Sublimate -	(41)	2 × 67	: 380	::Chlorine	44,1	: 125,4			
						10			
Protoxid - -	(42)	1 Oxygen + 2 Mercury				: 262	} Protoxid (w)		
Protoxid -	(43)	4	: 104	::Oxygen	10	: 260			
Calomel - -	(44)	11,5	: 88,5	::Mur. acid	34,1	: 262			

(22) Henry (23) by experiment (24) Biot and Arago (25) by Hypothesis (26) Gay Lussac
(27) Berthollet (28) Richter (29) by Hypothesis (30) R. Phillips (31) Thenard
(32) Berzelius (33) Gay Lussac (34) Chenevix (35) Proust (36) Marcet
(37) Davy (38) Wenzel (39) Fourcroy and Thenard. (40) Davy
(41) Davy (42) by Synthesis (43) Fourcroy (44) Chenevix

Water contained in crystallized Salts.

Sulphate of Copper	(45) 100	: 36,3	:: 156,6	: 56,8 =	5×11,36	
Sulphate of Iron	(46) 100	: 45,4	:: 173,8	: 79,0 =	7×11,28	
Sulphate of Zinc	(47) 100	: 44,3	:: 180,2	: 79,8 =	7×11,40	Water (w)
Sulphate of Magnesia	(48) 100	: 51,5	:: 153,9	: 79,3 =	7×11,33	
Glauber Salt	(49) 100	: 56	:: 202,3	: 113,1 =	10×11,31	
Muriate of Barytes	(50) 100	: 14,8	:: 153,6	: 22,8 =	2×11,42	

(45) Berzelius (46) Ditto (47) Lost by heat (48) By heat (49) Berzelius (50) Ditto.

In this table I have selected in most cases double evidence from different sources, in order that the inferences might receive confirmation from their concurrence. Number (29) may be noticed as a result anticipated from preceding data, and found to coincide with remarkable accuracy.

In the distillation of nitric acid from nitre, the whole of the acid may be obtained, if we employ enough of sulphuric acid to convert the residuum into bi-sulphate of potash. In this case each portion of potash, from which dry nitric acid is separated, will displace the water from two equivalent quantities of sulphuric acid, and each portion of nitric acid weighing 67,54 will be found combined with 22,64 of water. Hence 90,18 of liquid nitric acid so obtained should dissolve the equivalent 68 of carbonate of lime. And in fact, by an experiment carefully conducted on a large scale by Mr. PHILLIPS,* it appears that 681¼ of such acid did dissolve 476 of marble, which is in the proportion of 90,18 to 62,96, corresponding with the estimate within $\frac{1}{1100}$ part, a degree of coincidence rarely to be found even in the repetition of the same experiment by the most skilful analyst.

The specific gravity of this acid was found to be 1,50.

* Experimental Examination of the Pharm. Lond. by R. PHILLIPS.

Fig. 3.

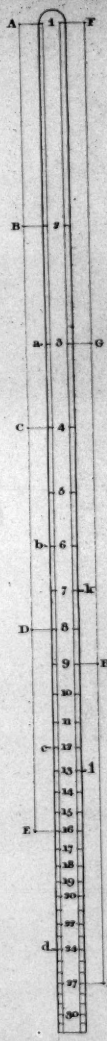


Fig. 2.

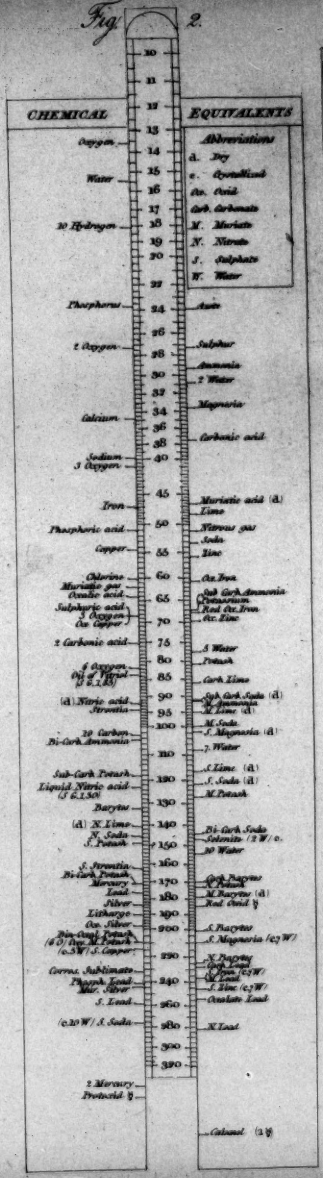
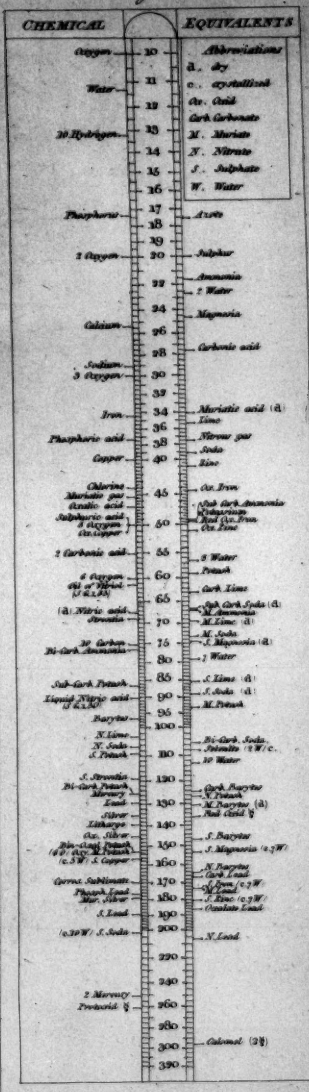


Fig. 1.



II. *Methods of clearing Equations of quadratic, cubic, quadrato-cubic, and higher Surds. By* William Allman, *M. D. Communicated by the Right Hon. Sir* Joseph Banks, *K. B. P. R. S.*

Read July 8, 1813.

Several years have elapsed, since my very highly esteemed friend, now Rev. Doctor Mooney, Fellow of Trinity College, Dublin, presented to the Royal Irish Academy a paper on the Extermination of Radicals from Equations. He has illustrated, by sundry examples, the extermination of quadratic surds. As he has rightly observed, the method is universal. Any number of quadratic surds, independent, or dependent, on each other, may be removed from an equation; because, 1. Any quantity, or factor of a quantity, necessarily subjected to the radical sign, is but of one dimension. 2. This quantity or factor being brought to one side of the equation, while the quantities unaffected with it remain at the other, may, by squaring both sides, be freed from the radical sign. 3. By a repetition of these reductions for each remaining independent surd quantity, any number of surd quantities may be converted into rational.

Examples.

I. Let $\sqrt{a}+\sqrt{b}+\sqrt{c}=o$: then

(2;) $\sqrt{a}+\sqrt{b}=-\sqrt{c}$; and, $a+b+2\sqrt{ab}=c$

(3;) $a+b-c=-2\sqrt{ab}$; $\overline{a+b-c}^{2}=4ab$; free from surds.

II. Let $\sqrt{a}+\sqrt{b}+\sqrt{c}+\sqrt{d}=0$; then

(2;) $a+b+c+2\sqrt{ab}+2\sqrt{ac}+2\sqrt{bc}=d$

(3;) $a+b+c-d+2\sqrt{ab}=-2\sqrt{ac}-2\sqrt{bc}$; squared gives, $\overline{a+b+c-d}^2+4ab+4.\overline{a+b+c-d}.\sqrt{ab}=4c.\overline{a+b+2\sqrt{ab}}$

(4;) $\overline{a+b-c-d}^2+4ab-4cd=4.\overline{c+d-a-b}.\sqrt{ab}$; squared, results free from surds.

Note, universally: the last two surd factors vanish together.

Surds, whose indices are integral powers of 2, may be treated as quadratic surds; and the number of them, which may be exterminated from any equation, is equally unlimited.

Let $\sqrt[4]{a}+\sqrt[4]{b}+\sqrt[4]{c}=0$: then, (2;) $\sqrt{a}+\sqrt{b}+2\sqrt[4]{ab}=\sqrt{c}$

(3;) $\sqrt{a}+\sqrt{b}-\sqrt{c}=-2\sqrt[4]{ab}$; and, $a+b+c+2\sqrt{ab}-2\sqrt{ac}-2\sqrt{bc}=4\sqrt{ab}$.

The surds, now all quadratic, may be thus exterminated:

(4;) $a+b+c-2\sqrt{ab}=2\sqrt{ac}+2\sqrt{bc}$; $\overline{a+b+c}^2+4a_b-4.\overline{a+b+c}.\sqrt{ab}=4ac+4bc+8c\sqrt{ab}$.

(5;) $\overline{a+b-c}^2+4ab=4.\overline{a+b+3c}.\sqrt{ab}$: put $a+b-c=2n$; then $n^2+ab=\overline{2n+4c}\sqrt{ab}$; and $n^4+2abn^2+a^2b^2=\overline{4n^2+16cn+16c^2}.ab$ $\therefore n^2-ab=\overline{16cn+16c^2}.ab$

i. e. $\frac{\overline{a+b-c}^2-4ab}{16}=8abc.\overline{2n+2c}=8abc.\overline{a+b+c}$

$\therefore \overline{a^2+b^2+c^2-2ab-2ac-2bc}^2=128abc.\overline{a+b+c}$.

Let $\sqrt[4]{a}+\sqrt[4]{b}+\sqrt[4]{c}=0$: then, by the last example,

(5;) $\overline{a+b+c-2\sqrt{ab}-2\sqrt{ac}-2\sqrt{bc}}^2=128.\overline{a\sqrt{bc}+b\sqrt{ac}+c\sqrt{ab}}$.

Put $a+b+c=2n$: then $\overline{n-\sqrt{ab}-\sqrt{ac}-\sqrt{bc}}^2$, or,

$$\begin{matrix} n^3 \\ +ab-2n \\ +ac+2a \\ +bc \end{matrix} \sqrt{bc} \begin{matrix} -2n. \\ +2b. \end{matrix} \sqrt{ac} \begin{matrix} -2n \\ +2c \end{matrix} . \sqrt{ab} = 32 . a\sqrt{bc} + b\sqrt{ac} + c\sqrt{ab}$$

$$(6;) \begin{matrix} n^3 \\ +ab-2n \\ +ac-30c \\ +bc \end{matrix} . \sqrt{ab} = \begin{matrix} 2n \\ +30b \end{matrix} . \sqrt{ac} \begin{matrix} +2n \\ +30a \end{matrix} . \sqrt{bc}$$

$$\overline{n^3+ab+ac+bc}^2 + 4ab . \overline{n+15c}^2 - 4 . \overline{n+15c} . \overline{n^3+ab+ac+bc}$$
$$. \sqrt{ab} = 4ac . \overline{n+15b}^2 + 4bc . \overline{n+15a}^2 + 8c . n+15b . n+15a . \sqrt{ab}$$

$$(7;) \overline{n^3+ab+ac+bc}^2 \begin{matrix} +4ab . \overline{n+15c}^2 \\ -4ac . \overline{n+15b}^2 \\ -4bc . n+15a \end{matrix} = 4 . \begin{matrix} \overline{n+15c . n^3+ab+ac+bc.} \\ +2c . n^3 \begin{matrix} +15a. \\ +15b. \end{matrix} +225ab \end{matrix} . \sqrt{ab}$$

$$\text{i.e. } n^6 \begin{matrix} +6ab. \\ -2ac. \\ -2bc. \end{matrix} n^3 - 120abcn \begin{matrix} +a^2b^2 \\ +a^2c^2 \\ +b^2c^2 \\ -898a^2bc \\ -898ab^2c \\ +902abc^2 \end{matrix} = 4 . n^3 + 17cn^2 \begin{matrix} +ab. \\ +31ac. \\ +31bc. \end{matrix} n \begin{matrix} +465abc \\ +15ac^2 \\ +15bc^2 \end{matrix} . \sqrt{ab},$$

which being squared, an equation will result of 8 dimensions, free from surds.

In like manner may surds of the 16th, 32d, 64th, &c. powers be taken away from any equation.

The number which may be taken away is unlimited, as the removal of each surd quantity or factor, in all these cases, depends on the principles which direct the solution of simple equations.

In the case of cubic surds, the quantity or factor necessarily subjected to the radical sign may be of one, or of two dimensions, but not higher: since then, an universal method is known for solving quadratic equations, any number of cubic surds, independent, or dependent, on each other, may be removed from an equation.

Let $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = o$: then $(2;)\ a + b + 3\sqrt[3]{a^2b} + 3\sqrt[3]{ab^2} = -c$: therefore, $(3;)\ \sqrt[3]{a^2b} + \sqrt[3]{ab^2} = \frac{-a-b-c}{3}$,

which put $= -m$: multiply by b: then, $\sqrt[3]{a^3b^3} + b\sqrt[3]{ab^3} = -bm$: this quadratic equation gives, $\sqrt[3]{ab^3} = -\frac{1}{2}b \pm \frac{1}{2}\sqrt{b^2 - 4bm}$ $\therefore ab^3 = \frac{-b^3+3b^2m}{2} \pm \frac{b^2-bm}{2}\sqrt{b^2-4bm}$; $2ab + b^2 - 3bm = \pm\overline{b-m}\sqrt{b^2-4bm}$: which squared gives, $4a^2b^2 + 4ab^3 - 12ab^2m + b^4 - 6b^3m + 9b^2m^2 = b^4 - 6b^3m + 9b^2m^2 - 4bm^3$ $\therefore a^2b + ab^2 - 3abm + m^2 = 0$; *i. e.* $m^3 - abc = 0$; $\therefore \overline{a+b+c}^3 = 27abc$.

But an equation consisting of 3 cubic surds only, may be cleared of them without the solution of a quadratic equation, thus:

Let $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$: then, (2;) $a + b + 3\sqrt[3]{a^2b} + 3\sqrt[3]{ab^2} = -c$, *i. e.* since $\sqrt[3]{a} + \sqrt[3]{b} = -\sqrt[3]{c}$, $a + b - 3\sqrt[3]{abc} = -c$: therefore, (3;) $a + b + c = 3\sqrt[3]{abc}$; and, $\overline{a+b+c}^3 = 27abc$.

And, universally, an equation consisting of 3 surds only, whose common index is any odd number, may be cleared of them, if we admit the solution of an equation, whose highest dimension is half the index of the surd minus unity.

Because in any integral power of a binomial, as the co-efficients of terms at equal distances from the extremes are equal, those terms may coalesce, the compound factor being equivalent to a simple one, as may be more fully seen below.

To return to cubic surds: if $s + t\sqrt[3]{d^2e} + v\sqrt[3]{de^2} = 0$, then, by the last example, (3;) $s^3 + t^3d^2e + v^3de^2 = 3stvde$.

Let $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{d} + \sqrt[3]{e} = 0$: then, (3;) $a + b + \sqrt[3]{d} + \sqrt[3]{e}^3 = 3\sqrt[3]{ab} \cdot \sqrt[3]{d} + \sqrt[3]{e}$;

and, $\overline{a + b + d + e + 3\sqrt[3]{d^2e} + 3\sqrt[3]{de^2}}^3 = 27ab \cdot \overline{d + e + 3\sqrt[3]{d^2e} + 3\sqrt[3]{de^2}}$: put $a + b + d + e = 3r$; then,

$$\overline{r + \sqrt[3]{d^2e} + \sqrt[3]{de^2}} = ab \cdot \overline{d + e} + 3\sqrt[3]{d^2e} + 3\sqrt[3]{de^2},$$

$$i.e.\ \begin{matrix} r^3 \\ +6der \\ +d^2s \\ +de^2 \end{matrix} \begin{matrix} +3r^2. \\ +3er. \\ +3dr. \\ +3de. \end{matrix} \sqrt[3]{d^2e} \begin{matrix} +3r. \\ +3dr. \\ +3de. \end{matrix} \sqrt[3]{de^2} = ab \cdot \overline{d + e} + 3ab^2\sqrt[3]{d^2e} + 3ab^2\sqrt[3]{de^2};$$

put $de - ab = x$, and $d + e = y$; $\therefore$ (4.) $\begin{matrix} r^3 \\ +6der \\ +xy \end{matrix} \begin{matrix} +3r^2. \\ +3er. \\ +3x. \end{matrix} \sqrt[3]{d^2e} \begin{matrix} +3r. \\ +3dr. \\ +3x. \end{matrix} \sqrt[3]{de^2} = 0.$

Substitute now, for s, $r^3 + 6der + xy$; for t, $3r^2 + 3er + 3x$; for v, $3r^2 + 3dr + 3x$; and the equation, $s^3 + t^3d^2e + v^3de^2 - 3stvde = 0$, will become,

$$r^9 - 9der^7 + 3xyr^6 \begin{matrix} +81d^2e^2. \\ -54dex. \end{matrix} r^5 \begin{matrix} -81d^2e^2y. \\ +63dx^2y. \end{matrix} r^4 \begin{matrix} +27d^2e^2y^2. \\ -27dex^3. \\ +3x^2y^2. \end{matrix} r^3 + 27dex^2yr^2$$
$$- 9dex^3y^3r + x^3y^3 = 0,$$

which is an equation of 9 dimensions, *i. e.* of 27 times the height of the given one $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{d} + \sqrt[3]{e} = 0$.

If another independent cubic surd were added, the equation resulting free from surds would be of 27 dimensions, or of 81 times the height of the given equation.

Universally, if an equation consist of any number of independent surds having a common index, the equation resulting free from surds will be so many times the height of the given one, as there are units in the common index of the surds raised to the power whose index is the number of independent surds diminished by unity.

As the solution of a cubic equation is required for the extermination of some of the higher surds, it may be worth while to shew the connection of the rule, called CARDAN's, with the extermination above given of cubic surds.

If $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$, then $a + b + c = 3\sqrt[3]{abc}$, as before; or, by substituting x, y, z, for $\sqrt[3]{a}$, $\sqrt[3]{b}$, $\sqrt[3]{c}$, respectively, if $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$. Suppose

any one of these quantities, as x, the unknown quantity of a cubic equation: arranged it may stand thus, $x^{3} * - 3yzx \overset{+y^3}{+z^3} = o$, an equation wanting the second term. Therefore, conversely, $x = -y - z$: or else, by dividing the cubic equation by the simple one $x \overset{+y}{+z} = o$, and solving the quote, the quadratic equation, $x^{2} \overset{-y}{-z}\, x \overset{+y^2}{-yz} = o$, $x = \frac{y+z}{2} \pm \frac{y-z}{2}\sqrt{-3}$. Then, in any cubic equation wanting the second term, *v. g.* $x^{3} * + qx + r = o$, suppose, $-3yz = q$; and, $y^3 + z^3 = r$: then, $z = -\frac{q}{3y}$; $z^3 = -\frac{q^3}{27y^3}$; and $y^3 - \frac{q^3}{27y^3} = r$: therefore, $y^6 = ry^3 + \frac{q^3}{27}$ $\therefore y^3 = \frac{1}{2}r \pm \sqrt{\frac{1}{4}r^2 + \frac{1}{27}q^3}$: and, by subtracting this from, $y^3 + z^3 = r$, $z^3 = \frac{1}{2}r \mp \sqrt{\frac{1}{4}r^2 + \frac{1}{27}q^3}$: therefore, $-y - z$, or $x = \sqrt[3]{-\frac{1}{2}r + \sqrt{\frac{1}{4}r^2 + \frac{1}{27}q^3}} + \sqrt[3]{-\frac{1}{2}r - \sqrt{\frac{1}{4}r^2 + \frac{1}{27}q^3}}$.

If the cubic equation have but one possible root, $\frac{y+z}{2} \pm \frac{y-z}{2}\sqrt{-3}$ will represent the two impossible roots. If the cubic equation have all its roots possible, the last named expression, as well as $-y - z$, implies the extraction of the cube root of an impossible binomial; except, however, in this single case, when two of the roots of the cubic equation are equal to each other: then, the solution by the above rule is possible, though all the roots be possible; for then, $y = z$; and the expressions $\frac{y-z}{2}$, and $\frac{1}{4}r^2 + \frac{1}{27}q^3$, both vanish: then $x^{3} * - 3y^2x + 2y^3 = o$, and the values of x, are $-2y$, y, and y.

Since any number, either of quadratic, or of cubic surds,

may be exterminated, any number of surds, whose indices are in any manner compounded of the factors 2 and 3, may also be exterminated from any equation.

It may at present suffice to instance, in surds of the 6th, and in surds of the 9th power.

Let $\sqrt[6]{a} + \sqrt[6]{b} + \sqrt[6]{c} = o$: then, as in cubic surds (3;) $\sqrt{a} + \sqrt{b} + \sqrt{c} = 3\sqrt[6]{abc}$; and $\overset{a.}{+3b.}\sqrt{a} \overset{+3a.}{+b.}\sqrt{b} \overset{+3a.}{+3b.}\sqrt{c} + 6\sqrt{abc} = 27\sqrt{abc}$. Put $3a + 3b + 3c = n$: then, (4;) $\overset{n.}{-2a.}\sqrt{a} \overset{+n.}{-2b.}\sqrt{b} = 21\sqrt{abc} \overset{-n.}{+2c.}\sqrt{c}$; and, by squaring $\overline{n^3 - 4an + 4a^2}.a + \overline{n^2 - 4bn + 4b^2}.b + 2n^2 \overset{-4a}{-4b}.n + 8ab\sqrt{ab} = 441abc + \overline{n^2 - 4cn + 4c^2}.c + \overline{84c^2 - 42cn}\sqrt{ab}$.

(5;) $\overset{a.}{+b.}n^2 \overset{-4a^2.}{-4b^2.}n \overset{+4a^2}{+4b^2}_{-4c^2} = -2n^2 \overset{+4a.}{+4b.}n \overset{-8ab}{+84c^2} \overset{}{-42c.}\sqrt{ab}$: by restitution, $\overset{a^3+15a^2b+15ab^2+b^3}{-3a^2c-423abc-3b^2c}_{+3ac^2+3bc^2} = \overline{-6a^2-20ab-6b^2}_{-150ac-150bc} \overset{}{-60c^2}\sqrt{ab}$. Put $a + b - c = r$.

Then $r^3 + 12abr - 405abc = \overline{-6r^2 - 162cr} \overset{-8ab}{-216c^2}\sqrt{ab}$.

Square and transpose, $r^6 - 12abr^4 - 2754abcr^3 \overset{+48a^2b^2.}{-28836abc^2.} \overset{-64a^3b^3}{r^2 - 123120a^2b^2c.} r + 1605692a^3b^3c^3 = 0$, results, an equation of 6 dimensions, free from surds. $\overset{}{-699840abc^3.}\overset{}{-466560abc^4}$

Let $\sqrt[9]{a} + \sqrt[9]{b} + \sqrt[9]{c} = o$; then, as in cubic surds (3;) $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 3\sqrt[9]{abc}$; and $\overset{a}{+b}\overset{}{+c} + 3\sqrt[3]{a^2b} + 3\sqrt[3]{ab^2} + 3\sqrt[3]{a^2c} + 6\sqrt[3]{abc} + 3\sqrt[3]{b^2c} + 3\sqrt[3]{ac^2} + 3\sqrt[3]{bc^2} = 27\sqrt[3]{abc}$.

Put $a+b+c=3n$: then

$(4;)\ \sqrt[3]{b}+\sqrt[3]{c}\,.\,\sqrt[3]{a^2}+\sqrt[3]{b^2}-\sqrt[3]{7bc}+\sqrt[3]{c^2}\,.\,\sqrt[3]{a}=-\sqrt[3]{b^2c}-\sqrt[3]{bc^2}-n$: multiply by $\sqrt[3]{b}+\sqrt[3]{c}$: then $\sqrt[3]{b}+\sqrt[3]{c}\,.\,\sqrt[3]{a^2}+\sqrt[3]{b^2}-\sqrt[3]{7bc}+\sqrt[3]{c^2}\,.\,\sqrt[3]{a}=-\sqrt[3]{bc}\,.\,\sqrt[3]{b}+\sqrt[3]{c}-n\,.\,\sqrt[3]{b}+\sqrt[3]{c}$: this quadratic equation, when solved, gives

$$\sqrt[3]{b}+\sqrt[3]{c}\,.\,\sqrt[3]{a}=\frac{-\sqrt[3]{b^2}+7\sqrt[3]{bc}-\sqrt[3]{c^2}}{2}\pm\frac{\sqrt{b-18c-4n\,.\,\sqrt[3]{b}+43\sqrt[3]{b^2c^2}+c-18b-4n\,.\,\sqrt[3]{c}}}{2}.$$

This equation, when cubed, will exhibit the quantity or factor a, free from the cubic radical sign; thus, $a\,.\,\sqrt[3]{b}+\sqrt[3]{c}^3$; or,

$$+\overset{ab}{ac}+3a^2\sqrt[3]{bc}+3a\sqrt[3]{bc}^2=\tfrac{1}{2}\,.\,\overset{-b^2}{\underset{-c^2}{\underset{+3bn}{\underset{+3cn}{+34bc+24b.}}}}\overset{-165b.}{\underset{-18n.}{-165c.\sqrt[3]{bc}+24c.}}\sqrt[3]{bc}^2$$

$$\pm\tfrac{1}{2}\,.\,\overset{b.}{\underset{-n.}{-15c.\sqrt[3]{b}+43\sqrt[3]{bc}^2}}\overset{-15b.}{\underset{-n.}{+c.\sqrt[3]{c}}}\sqrt{\overset{b.}{\underset{-42n.}{-18c.\sqrt[3]{b}+43\sqrt[3]{bc}^2}}\overset{-18b.}{\underset{-4n.}{+c.\sqrt[3]{c}}}}.$$

Restore for n, $\frac{a+b+c}{3}$, transpose, and double: then

$$-351bc-\overset{ab+12a.}{18b.\sqrt[3]{bc}+171b.\sqrt[3]{bc}^2}=\pm\overset{\frac{1}{3}a.}{\underset{-\frac{46}{3}c.}{+\frac{2}{3}b.\sqrt[3]{b}+43\sqrt[3]{bc}^2}}\overset{-\frac{1}{3}a.}{-\frac{46}{3}b.\sqrt[3]{c}}$$
$$+\overset{+12a.}{\underset{-18c.}{ac+171c.}}\qquad\qquad\qquad

$$\begin{array}{l} b. \\ +c. \end{array} \begin{array}{l} a^3 \\ \end{array} \begin{array}{l} +3b^3. \\ +15\,18bc. \\ +3c^3. \end{array} \begin{array}{l} a^2 \\ \end{array} \begin{array}{l} +3b^3. \\ +3168b^2c. \\ +3168bc^2. \\ +3c^3. \end{array} \begin{array}{l} a \\ \end{array} \begin{array}{l} +b^4. \\ +2208b^3c \\ +9248b^2c^2 \\ +2208bc^3 \\ +c^4. \end{array} + 3 \cdot \begin{array}{l} a^3 \\ \end{array} \begin{array}{l} +87b. \\ +339c. \end{array} \begin{array}{l} a^2 \\ \end{array} \begin{array}{l} -1536^2. \\ -3357bc. \\ +1089c^2. \end{array}$$

$$a \begin{array}{l} +4b^2 \\ +1656^2c \\ +264bc^2 \\ +22c^2 \end{array} \cdot \sqrt[3]{b^2c} + 3 \cdot \begin{array}{l} a^3 \\ \end{array} \begin{array}{l} +339b. \\ +87c. \end{array} \begin{array}{l} a^2 \\ \end{array} \begin{array}{l} +19898^2. \\ -3357bc. \\ -153c^2 \end{array} a \begin{array}{l} +22b^2 \\ +264b^2c \\ +165bc^2 \\ +4c^3 \end{array} \cdot \sqrt[3]{bc^2} = o.$$

Compare this with the equation, $s + t\sqrt[3]{d^2e} + v\sqrt[3]{de^2} = o$: the resulting equation, $s^3 + 3s^2d^2e + v^3d^4e^4 = 3stvde$, being accordingly computed, will be free from surds. It will be of 12 dimensions; but may be depressed to one of 9. Instead of continuing the operation to shew this, I refer to the extermination of surds of the 5th, and of the 7th power, to be given below, for the manner in which some equations, resulting on involution, are depressed.

In surds of the 5th power, the quantity or factor, necessarily subjected to the radical sign, may be of 4 dimensions, but not higher: whence, if the solution of any biquadratic equation be admitted, any number of surds of the 5th power may be taken away from an equation; and here it may be observed, that, as to the matter in hand, it is of no importance, whether the biquadratic equation may be solved in possible terms, or not; for the value, in numbers, of any particular quantity, or factor, is not required; it is only required to obtain the quantity, or factor, of a single dimension, in order to deprive it, by involution, of its radical sign.

When an equation consists of 3 surds of the 5th power, the biquadratic equation is virtually a quadratic.

Let $\sqrt[5]{a} + \sqrt[5]{b} + \sqrt[5]{c} = o \therefore (2;)\ a + b + 5\sqrt[5]{a^2b} + 10\sqrt[5]{a^3b^2} + 10\sqrt[5]{a^2b^3} + 5\sqrt[5]{ab^4} = -c$; put $a + b + c = 5m$. $\therefore (3;)\ \sqrt[5]{a^4b} + 2\sqrt[5]{a^3b^2} + 2\sqrt[5]{a^2b^3} + \sqrt[5]{ab^4} = -m$: this,

multiplied by b^2, gives $\sqrt[5]{a^4b^6} + 2b\sqrt[5]{a^3b^{11}} + 2b^2\sqrt[5]{a^2b^8} + b^3\sqrt[5]{ab^5} = -b^3m$, a biquadratic equation of the quadratic form, thus, $\sqrt[5]{a^2b^8} + b^3\sqrt[5]{ab^5} + b^3 \cdot \sqrt[5]{a^2b^8} + b^3\sqrt[5]{ab^5} = -b^3m$, which solved, gives $\sqrt[5]{a^2b^8} + b\sqrt[5]{ab^4} = -\frac{b^3}{2} \pm \frac{\sqrt{b^6 - 4b^3m}}{2}$, another quadratic: then $\sqrt[5]{ab^4} = -\frac{b}{2} \pm \frac{\sqrt{-b^3 \pm 2\sqrt{b^6 - 4b^3m}}}{2}$; involve to the 5th power, and the equation will be cleared of the radical sign of the surd of the 5th power: thus,

$$ab^4 = -\tfrac{1}{2}b^5 + \tfrac{5}{2}b^4m + \tfrac{1}{2}b^3\sqrt{b^6 - 4b^3m} \mp \tfrac{1}{2}b^3m\sqrt{-b^3 \pm 2\sqrt{b^6 - 4b^3m}}$$

divide by $\frac{1}{2}b^3$, and transpose; then,

$$b^2 - 5bm + 2ab = \sqrt{b^6 - 4bm} \mp m\sqrt{-b^3 \pm 2b\sqrt{b^6 - 4bm}}.$$

Square, transpose, and divide by $2b$; then,

$$b^3 \begin{matrix} -5m. \\ +2a. \end{matrix} b^2 \begin{matrix} +5m^2. \\ -10am. \\ +2a^2. \end{matrix} b = \pm\sqrt{b^6 - 8bm + m^2}\sqrt{b^6 - 4bm}.$$

Square again, transpose, divide by $4b$, and arrange:

then, $m^5 * - 25abm^3 \begin{matrix} +30a^2b. \\ +30ab^2. \end{matrix} m^2 \begin{matrix} -10a^3b. \\ -15a^2b^2. \\ -10ab^3. \end{matrix} m \begin{matrix} +a^4b \\ +2a^3b^2 \\ +2a^2b^3 \\ +ab^4 \end{matrix} = 0.$

But, as in the case of 3 cubic surds, a simple equation supplied the place of a quadratic, so, when an equation consists of 3 surds of the 5th power, a quadratic may supply the place of a biquadratic, or of two quadratic equations.

Thus, $\sqrt[5]{a} + \sqrt[5]{b} + \sqrt[5]{c} = o$, $\because$ (2;) $a + b + 5\sqrt[5]{a^4b} + 10\sqrt[5]{a^3b^2} + 10\sqrt[5]{a^2b^3} + 5\sqrt[5]{ab^4} = -c$, (3;) $\sqrt[5]{a^4b} + 2\sqrt[5]{a^3b^2} + 2\sqrt[5]{a^2b^3} + \sqrt[5]{ab^4} = \frac{-a-b-c}{5} = -m$:

But $\left\{ \begin{matrix} \sqrt[5]{a^4b} + 3\sqrt[5]{a^3b^2} + 3\sqrt[5]{a^2b^3} + \sqrt[5]{ab^4} = \sqrt[5]{ab} \cdot \sqrt[5]{a} + \sqrt[5]{b}^3 & = \sqrt[5]{ab} \cdot -\sqrt[5]{c}^3 = -\sqrt[5]{abc^3} \\ -\sqrt[5]{a^3b^2} - \sqrt[5]{a^2b^3} & = -\sqrt[5]{a^2b^3} \cdot \sqrt[5]{a} + \sqrt[5]{b} = -\sqrt[5]{a^2b^3} \cdot -\sqrt[5]{c} = +\sqrt[5]{a^2b^2c} \end{matrix} \right\}$

Therefore $\sqrt[5]{a^2b^2c} - \sqrt[5]{abc^3} = -m.$

This, multiplied by c, gives the quadratic equation, $\sqrt[5]{a^2b^2c^5}$

$-c\sqrt{abc^3} = -cm$: which solved gives $\sqrt{abc^3} = \frac{c \pm \sqrt{c^2 - 4cm}}{2}$: involve to the 5th power; then, $abc^3 = \frac{c^5 - 5c^4m + 5c^3m^2}{2} \pm \frac{c^4 - 3c^3m + c^2m^2}{2}\sqrt{c^2 - 4cm}$.

Divide by $\frac{1}{2}c^2$, and transpose; then, $-c^3 + 5mc^2 \overset{-5m^2.}{+2ab.} c = \pm \sqrt{c^2 - 3mc + m^2}\sqrt{c^2 - 4mc}$: squared, gives

$c^6 - 10mc^5 \overset{+35m^2.}{-4ab.} c^4 \overset{-50m^3.}{+20abm.} c^3 \overset{+25m^4.}{-20abm^2.} \overset{}{+4a^2b^2.} c^3 = c^6 - 10mc^5 + 35m^2c^4 - 50m^3c^3 + 25m^4c^2 - 4m^5c$.

Transpose, divide by $4c$, and arrange; then, $m^5 * * - 5abcm^3 + 5abc^3m \overset{+a^2b^2c}{-abc^3} = 0$; or, $m^3 = abc \cdot \overline{5m^2 - ab - ac - bc}$; which is an equation of 5 dimensions, free from surds.

This equation, if, instead of $\sqrt[1]{a} + \sqrt[1]{b} + \sqrt[1]{c} = 0$, were substituted, $\sqrt[10]{a} + \sqrt[10]{b} + \sqrt[10]{c} = 0$, would contain no other than quadratic surds; if, $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$, no higher than cubic surds; wherefore, if the extermination of any number of surds of the 5th power from an equation be admitted, since the number of surds of any lower order which may be exterminated is unlimited, an equation consisting of any number of surds, whose indices are in any manner compounded of the factors, 2, 3, and 5, may be totally freed from surds.

If a formula for the solution of any equation of 6 dimensions were known, any number of surds of the 7th power might be taken away from an equation: As such a formula, however, is, I suppose, at present altogether unknown, we may be contented with the extermination of 3 surds of the 7th power, which may be accomplished, because, a formula for the solution of cubic equations is known, 3, the index of

the cube, being equal to half the index of the 7th power diminished by unity.

Let $\sqrt[7]{a}+\sqrt[7]{b}+\sqrt[7]{c}=o$: then,

$$(2;)\ a+b+7\sqrt[7]{a^6b}+21\sqrt[7]{a^5b^2}+35\sqrt[7]{a^4b^3}+35\sqrt[7]{a^3b^4}+21\sqrt[7]{a^2b^5}+7\sqrt[7]{ab^6}=-c$$

$$(3;)\ \sqrt[7]{a^6b}+3\sqrt[7]{a^5b^2}+5\sqrt[7]{a^4b^3}+5\sqrt[7]{a^3b^4}+3\sqrt[7]{a^2b^5}+\sqrt[7]{ab^6}=\frac{-a-b-c}{7}\ (\text{put})=-m$$

But
$$\left\{\begin{array}{ll} \sqrt[7]{a^6b}+5\sqrt[7]{a^5b^2}+10\sqrt[7]{a^4b^3}+10\sqrt[7]{a^3b^4}+5\sqrt[7]{a^2b^5}+\sqrt[7]{ab^6}=\sqrt[7]{ab}.\overline{\sqrt[7]{a}+\sqrt[7]{b}}^5=-\sqrt[7]{abc^5} & \\ -2\sqrt[7]{a^5b^2}-6\sqrt[7]{a^4b^3}-6\sqrt[7]{a^3b^4}-2\sqrt[7]{a^2b^5} & =-2\sqrt[7]{a^2b^2}.\overline{\sqrt[7]{a}+\sqrt[7]{b}}^3=+2\sqrt[7]{a^2b^2c^3} \\ +\sqrt[7]{a^4b^3}+\sqrt[7]{a^3b^4} & =+\sqrt[7]{a^3b^3}.\overline{\sqrt[7]{a}+\sqrt[7]{b}}=-\sqrt[7]{a^3b^3c} \end{array}\right\}$$

Therefore $-\sqrt[7]{a^3b^3c}+2\sqrt[7]{a^2b^2c^2}-\sqrt[7]{abc^5}=-m.$

Multiply by $-c^3$; then, $\sqrt[7]{a^3b^3c^{15}}-2c\sqrt[7]{a^2b^2c^{10}}+c^3\sqrt[7]{abc^5}=c^3m$. Extract the square root of this cubic equation; then, $\sqrt[14]{a^3b^3c^{15}}*-c\sqrt[14]{abc^5}=\pm c\sqrt{m}$: this cubic equation, which wants the second term, when solved, gives $\sqrt[14]{abc^5}=\sqrt[3]{\pm\frac{1}{2}c\sqrt{m}:+\frac{1}{2}c\sqrt{m-\frac{4}{27}c}}+\sqrt[3]{\pm\frac{1}{2}c\sqrt{m}:-\frac{1}{2}c\sqrt{m-\frac{4}{27}c}}$: this, involved to the 14th power, will be free from all surds, except quadratic and cubic.

Put $m-\frac{4}{27}c=n$, $\pm\frac{1}{2}\sqrt{m}:+\frac{1}{2}\sqrt{n}=s$, and $\pm\frac{1}{2}\sqrt{m}:-\frac{1}{2}\sqrt{n}=t$: then, on dividing the equation by $\sqrt[3]{c}$, $\frac{\sqrt[14]{abc^5}}{\sqrt[3]{c^2}}=\sqrt[3]{s}+\sqrt[3]{t}$: involve, then $\frac{abc}{\sqrt[3]{c^2}}=\sqrt[3]{s^{14}}+14\sqrt[3]{s^{13}t}+91\sqrt[3]{s^{12}t^2}+364\sqrt[3]{s^{11}t^3}+1001\sqrt[3]{s^{10}t^4}+2002\sqrt[3]{s^9t^5}+3003\sqrt[3]{s^8t^6}+3432\sqrt[3]{s^7t^7}+3003\sqrt[3]{s^6t^8}+2002\sqrt[3]{s^5t^9}+1001\sqrt[3]{s^4t^{10}}+364\sqrt[3]{s^3t^{11}}+91\sqrt[3]{s^2t^{12}}+14\sqrt[3]{st^{13}}+\sqrt[3]{t^{14}}$.

But, $st=\frac{1}{4}m-\frac{1}{4}n=\frac{1}{27}c$ $\therefore\frac{1}{3}\sqrt[3]{c}=\sqrt[3]{st}$, and $\frac{1}{9}\sqrt[3]{c^2}=\sqrt[3]{s^2t^2}$: then, by multiplication, $\frac{1}{9}abc=s^3\sqrt[3]{st^2}+14s^2t\sqrt[3]{s^2t}+364s^4\sqrt[3]{st^2}+1001s^4t^2+2002s^3t^3\sqrt[3]{s^2t}+3003s^3t^2\sqrt[3]{st^2}+3432s^2t^3+3003s^2t^3\sqrt[3]{s^2t}+2002s^2t^3\sqrt[3]{st^2}+1001s^2t^4+364st^4\sqrt[3]{s^2t}+91st^4\sqrt[3]{st^2}+14st^5+t^5\sqrt[3]{s^2t}$

$$\begin{array}{rrrr} & 14s^3l & + 91s^2l. & + s^3. \\ +10015^4l^3 & + 2002s^3l^2. & + 364s^2l. & \\ \end{array}$$

Arranged, $\frac{1}{5}abc = +343s^2l^3 + 3003s^2l^3.\sqrt[3]{s^3l} + 3003s^3l^2.\sqrt[3]{sl^2}$
$+10015s^4l^2 + 364sl^2. \quad +2002s^2l^2.$
$+ 14sl^3 + l^3. \quad +91sl^2.$

Restore, first, for s, $\pm\frac{1}{2}\sqrt{m}$: $+\frac{1}{2}\sqrt{n}$; for l, $\pm\frac{1}{2}\sqrt{m}$: $-\frac{1}{2}\sqrt{n}$: then,

$$\frac{1}{5}abc = \frac{1}{32}\times\left(\begin{matrix}2731m^3.\\-3348mn.\\+729n^2.\end{matrix}\overline{m-n}\sqrt{\begin{matrix}+5461m^3.\\-9090mn.\\+3645n^2.\end{matrix}\pm\sqrt{m}\begin{matrix}-1825m^2.\\+2538mn.\\-729n^2.\end{matrix}\sqrt{n}}.\right.$$

$$\frac{1}{2}\sqrt[3]{\begin{matrix}m.\\-n.\end{matrix}\pm\sqrt{m}\begin{matrix}+m.\\-n.\end{matrix}\sqrt{n}}\sqrt{\begin{matrix}+5461m^3.\\-9090mn.\\+3645n^2.\end{matrix}\pm\sqrt{m}\begin{matrix}+1825m^2.\\-2538mn.\\+729n^2.\end{matrix}\sqrt{n}}.$$

$$\left.\frac{1}{2}\sqrt[3]{\begin{matrix}m.\\-n.\end{matrix}\pm\sqrt{m}\begin{matrix}-m.\\+n.\end{matrix}\sqrt{n}}\right).$$

Restore, secondly, for n, $m-\frac{4}{27}c$; divide by $\frac{1}{2}\sqrt[3]{c}$, and transpose: then,

$$\begin{matrix}14m^3.\\+35cm.\\+2c^3.\\-3ab.\end{matrix}\frac{1}{9}\sqrt[3]{c^3}\sqrt{\begin{matrix}+m^3.\\+\frac{50}{3}cm.\\+5c^3.\end{matrix}\pm\frac{1}{2}\sqrt{m}\begin{matrix}-m^3.\\-10cm.\\-c^3.\end{matrix}\frac{1}{2}\sqrt{m-\frac{4}{27}c}}$$

$$\sqrt[3]{\pm\frac{1}{2}\sqrt{m}+\frac{1}{2}\sqrt{m-\frac{4}{27}c}}\sqrt{\begin{matrix}+m^3.\\+\frac{50}{3}cm.\\+5c^3.\end{matrix}\pm\frac{1}{2}\sqrt{m}\begin{matrix}+m^3.\\+10cm.\\+c^3.\end{matrix}\frac{1}{2}\sqrt{m-\frac{4}{27}c}}$$

$$\sqrt[3]{\pm\frac{1}{2}\sqrt{m}-\frac{1}{2}\sqrt{m-\frac{4}{27}c}}=0.$$

Put, $m^3+\frac{50}{3}cm+5c^2=2x$; $m^2+10cm+c^2=2y$; $14m^3+35cm+2c^2-3ab=9z$: then, $\overline{\pm x\sqrt{m}-y\sqrt{n}}.\sqrt[3]{\pm\frac{1}{2}\sqrt{m}+\frac{1}{2}\sqrt{n}}+\overline{\pm x\sqrt{m}+y\sqrt{n}}.\sqrt[3]{\pm\frac{1}{2}\sqrt{m}-\frac{1}{2}\sqrt{n}}+z\sqrt[3]{c^2}=0.$

Therefore, by the extermination of cubic surds,

$$\begin{cases}\overline{\pm x\sqrt{m}-y\sqrt{n}}^3.\pm\frac{1}{2}\sqrt{m}+\frac{1}{2}\sqrt{n}\\+\overline{\pm x\sqrt{m}+y\sqrt{n}}^3.\pm\frac{1}{2}\sqrt{m}-\frac{1}{2}\sqrt{n}+cz^3=3.\overline{x^2m-y^2n}.\\\sqrt[3]{\frac{1}{4}m-\frac{1}{4}n}.z\sqrt[3]{c^2}=\overline{x^2m-y^2n}.cz\end{cases}$$

i. e. $x^3m^3 - 3x^2ymn + 3xy^2mn - y^3n^3 + c^3z^3 = \overline{x^3m - y^3n} \cdot cz$;
or, $x^3m + 3y^3n \cdot xm - : 3x^3m + y^3n \cdot yn - : \overline{x^3m - y^3n - cz^3} \cdot cz = 0$.
Restore the values of x, y, and z: divide by $\frac{1}{27}c$; then,

$$m^7 * - 25c^3m^3 \begin{matrix} -\frac{14600}{27}c^3. \\ -35abc. \end{matrix} m^4 \begin{matrix} -\frac{7850}{27}c^4. \\ +98abc^2. \end{matrix} m^3 \begin{matrix} -\frac{200}{27}c^3. \\ -\frac{164}{3}abc^3. \\ +14a^2b^2c. \end{matrix} m^3 \begin{matrix} +14abc^4. \\ +35a^2b^2c^2. \end{matrix}$$
$$m \begin{matrix} -abc^5 \\ +2a^2b^2c^3 \\ +a^3b^3c \end{matrix} = 0$$: which is an equation of 7 dimensions, free from surds.

In like manner, the extermination of 3 surds of the 11th power from an equation might seem to require the solution of an equation of 5 dimensions: but in this case, the highest term, if I may so speak, vanishes; so that an equation, consisting of 3 surds of the 11th power, may be freed from surds, without the solution of any higher equation than a biquadratic. The labour however is great.

As preparatory thereto, and not to refer elsewhere, the solution of a biquadratic equation may be here given.

Suppose it, as any equation may be so transformed, to want the second term; thus,
$x^4 * + qx^2 + rx + s = 0$: suppose also, $x^2 + ex + f = 0$;
$x^2 - ex + g = 0$: then,

$$x^4 * \begin{matrix} +f. \\ +g. \\ -e^2. \end{matrix} x^2 \begin{matrix} -ef. \\ +eg. \end{matrix} x + fg = 0$$: and, $\left.\begin{matrix} f+g-e^2 = q \\ -ef+eg = r \\ fg = s \end{matrix}\right\}$

Therefore, $\left.\begin{matrix} f+g = e^2+q \\ -f+g = \frac{r}{e} \end{matrix}\right\} \begin{matrix} f = \frac{e^3+qe-r}{2e} \\ g = \frac{e^3+qe+r}{2e} \end{matrix} \Big\} fg = \frac{e^6+2qe^4+q^2e^2-r^2}{4e^2} = s$;

then, $e^6 + 2qe^4 \begin{matrix} +q^2. \\ -4s. \end{matrix} e^2 - r^2 = 0$. Put $e^2 = z - \frac{2}{3}q$; then,

$$\left.\begin{array}{rrr} z^3 - 2qz^2 + \frac{4}{3}q^2z - & \frac{8}{27}q^3 = e^3 & \\ + 2qz^2 - \frac{8}{3}q^2z + & \frac{8}{9}q^3 = + 2qe^2 & \\ & -\frac{2}{3}q^2 & \\ q^2 \cdot z & + \frac{8}{3}qs = +q^2 \cdot e^2 & \\ -4s. & & -4s. \\ & -r^2 = -r^2 & \end{array}\right\} \therefore z^3 \overset{-\frac{1}{3}q^2.}{-4s.} z \overset{-\frac{2}{27}q^3}{+\frac{8}{3}qs} = 0.$$

This cubic equation gives, $z = \sqrt[3]{\frac{1}{27}q^3 - \frac{4}{3}qs + \frac{1}{2}r^2 + \frac{1}{3}\sqrt{-\frac{4}{3}q^2s + \frac{1}{3}q^2r^2 + \frac{32}{3}q^2s^2 - 12qr^2s + \frac{9}{4}r^4 - \frac{64}{3}s^3}} + \sqrt[3]{\frac{1}{27}q^3 - \frac{4}{3}qs + \frac{1}{2}r^2 - \frac{1}{3}\sqrt{-\frac{4}{3}q^2s + \frac{1}{3}q^2r^2 + \frac{32}{3}q^2s^2 - 12qr^2s + \frac{9}{4}r^4 - \frac{64}{3}s^3}}$.

Then by hypothesis and solution of quadratics, $x = -\frac{1}{2}e \pm \sqrt{\frac{1}{4}e^2 - f}$: or, $x = +\frac{1}{2}e \pm \sqrt{\frac{1}{4}e^2 - g}$; and, by substitution, $x = -\frac{1}{2}e \pm \frac{1}{2}\sqrt{-e^2 - 2q + 2\frac{r}{e}} = \mp\frac{1}{2}\sqrt{z - \frac{2}{3}q} \pm \frac{1}{2}\sqrt{-z - \frac{4}{3}q + \frac{2r}{\sqrt{z - \frac{2}{3}q}}}$, or $x = \frac{1}{2}e \pm \frac{1}{2}\sqrt{-e^2 - 2q - 2\frac{r}{e}} = \pm\frac{1}{2}\sqrt{z - \frac{2}{3}q} \pm \frac{1}{2}\sqrt{-z - \frac{4}{3}q - \frac{2r}{\sqrt{z - \frac{2}{3}q}}}$.

Let now the equation consisting of 3 surds of the 11th power be, $\sqrt[11]{a} + \sqrt[11]{b} + \sqrt[11]{c} = o$; then

(2;) $a + b + 11\sqrt[11]{a^{10}b} + 55\sqrt[11]{a^9b^2} + 165\sqrt[11]{a^8b^3} + 330\sqrt[11]{a^7b^4} + 462\sqrt[11]{a^6b^5} + 462\sqrt[11]{a^5b^6} + 330\sqrt[11]{a^4b^7} + 165\sqrt[11]{a^3b^8} + 55\sqrt[11]{a^2b^9} + 11\sqrt[11]{ab^{10}} = -c$.

$$
\begin{aligned}
&\text{(33)}\quad x^{12}/a^{12} + \tfrac{5}{6}x^{11}/a^{11} + \tfrac{5}{3}x^{10}/a^{10} + \tfrac{10}{9}x^{9}/a^{9} + \tfrac{35}{36}x^{8}/a^{8} + \tfrac{7}{18}x^{7}/a^{7} + \tfrac{7}{72}x^{6}/a^{6} + \tfrac{1}{72}x^{5}/a^{5} + \tfrac{1}{288}x^{4}/a^{4} \\
&\quad - \tfrac{1}{12}x^{11}/a^{11}b + \tfrac{1}{3}x^{10}/a^{10}b + \tfrac{1}{3}x^{9}/a^{9}b + \tfrac{1}{6}x^{8}/a^{8}b + \tfrac{1}{12}x^{7}/a^{7}b + \tfrac{1}{36}x^{6}/a^{6}b + \tfrac{1}{72}x^{5}/a^{5}b + \tfrac{1}{288}x^{4}/a^{4}b \\
&\quad + \tfrac{1}{12}x^{10}/a^{10}b^{2} - \tfrac{1}{6}x^{9}/a^{9}b^{2} - \tfrac{1}{6}x^{8}/a^{8}b^{2} - \tfrac{1}{12}x^{7}/a^{7}b^{2} - \tfrac{1}{36}x^{6}/a^{6}b^{2} - \tfrac{1}{72}x^{5}/a^{5}b^{2} - \tfrac{1}{288}x^{4}/a^{4}b^{2} \\
&\quad - \tfrac{1}{12}x^{9}/a^{9}b^{3} + \tfrac{1}{6}x^{8}/a^{8}b^{3} + \tfrac{1}{12}x^{7}/a^{7}b^{3} + \tfrac{1}{36}x^{6}/a^{6}b^{3} + \tfrac{1}{72}x^{5}/a^{5}b^{3} + \tfrac{1}{288}x^{4}/a^{4}b^{3} \\
&\quad + \tfrac{1}{12}x^{8}/a^{8}b^{4} - \tfrac{1}{12}x^{7}/a^{7}b^{4} - \tfrac{1}{36}x^{6}/a^{6}b^{4} - \tfrac{1}{72}x^{5}/a^{5}b^{4} - \tfrac{1}{288}x^{4}/a^{4}b^{4} \\
&\quad - \tfrac{1}{12}x^{7}/a^{7}b^{5} + \tfrac{1}{36}x^{6}/a^{6}b^{5} + \tfrac{1}{72}x^{5}/a^{5}b^{5} + \tfrac{1}{288}x^{4}/a^{4}b^{5} \\
&\quad + \tfrac{1}{36}x^{6}/a^{6}b^{6} - \tfrac{1}{72}x^{5}/a^{5}b^{6} - \tfrac{1}{288}x^{4}/a^{4}b^{6} \\
&\quad - \tfrac{1}{72}x^{5}/a^{5}b^{7} + \tfrac{1}{288}x^{4}/a^{4}b^{7} \\
&\quad + \tfrac{1}{288}x^{4}/a^{4}b^{8} \\
&= m
\end{aligned}
$$

Therefore * $5\sqrt[11]{a^4b^6c^3} - 7\sqrt[11]{a^3b^3c^3} + 4\sqrt[11]{a^2b^6c^3} - \sqrt[11]{abc^9} = -m\ (= \frac{-a-b-c}{11})$ Multiply by $\frac{1}{5}c^3$; $\sqrt[11]{a^4b^6c^{25}} - \frac{7}{5}c\sqrt[11]{a^3b^3c^{27}} + \frac{4}{5}c^2\sqrt[11]{a^2b^6c^{28}} - \frac{1}{5}c^3\sqrt[11]{abc^9} = -\frac{1}{5}c^3m.$

Put, $\sqrt[11]{abc^9} = x + \frac{7}{20}c$; then,

$$\left.\begin{array}{r} x^4 + \frac{7}{5}cx^3 + \frac{147}{200}c^2x^2 + \frac{343}{2000}c^3x + \frac{2401}{160000}c^4 \\ -\frac{7}{5}cx^3 - \frac{147}{100}c^2x^2 - \frac{1029}{2000}c^3x - \frac{2401}{40000}c^4 \\ +\frac{4}{5}c^2x^2 + \frac{14}{25}c^3x + \frac{49}{500}c^4 \\ -\frac{1}{5}c^3x - \frac{7}{100}c^4 \\ +\frac{1}{5}c^3m \end{array}\right\} \quad \left.\begin{array}{l} x^4 * + \frac{13}{200}c^2x^2 + \frac{17}{1000} \\ c^3x \quad -\frac{2723}{160000}c^4 \\ \quad +\frac{1}{5}c^3m \end{array}\right\} = 0.$$

To solve this biquadratic equation, substitute in the equation, $z^3 * \begin{matrix} -\frac{1}{13}q^3 \\ -\frac{4}{13}r^3 \end{matrix} z + \begin{matrix} -\frac{2}{13}rq^2 \\ -\frac{1}{13}q^2s \end{matrix} = 0$, for q, $\frac{13}{200}c^2$; for r, $\frac{17}{1000}c^3$; and for s, $-\frac{2723}{160000}c^4 + \frac{1}{5}c^3m$: then, $z^3 * \begin{matrix} +\frac{1}{13}c^4 \\ -\frac{1}{13}c^3m \end{matrix} z \begin{matrix} -\frac{1}{13\cdot13}c^5 \\ +\frac{13}{13\cdot13}c^5m \end{matrix} = 0$:

$$\therefore z = \tfrac{1}{13}c \sqrt[3]{\tfrac{11}{2}c^3 - \tfrac{117}{2}c^3m + \tfrac{1}{2}c\sqrt{13621c^4 - 164574c^3m + 661689c^2m^2 - 864000cm^3}} + \tfrac{1}{13}c \sqrt[3]{\tfrac{11}{2}c^3 - \tfrac{117}{2}c^3m - \tfrac{1}{2}c\sqrt{13621c^4 - 164574c^3m + 661689c^2m^2 - 864000cm^3}}$$

then, $x = \mp\frac{1}{2}\sqrt{z - \frac{13}{300}c^2} \pm \frac{1}{2}\sqrt{-z - \frac{13}{150}c^2 + \frac{\frac{17}{300}c^3}{\sqrt{z - \frac{13}{300}c^2}}}$:

or, $x = \pm\frac{1}{2}\sqrt{z - \frac{13}{300}c^2} \pm \frac{1}{2}\sqrt{-z - \frac{13}{150}c^2 - \frac{\frac{17}{300}c^3}{\sqrt{z - \frac{13}{300}c^2}}}$

$$\left.\begin{array}{l} \therefore \sqrt[11]{abc^9} = \frac{7}{20}c \mp \frac{1}{2}\sqrt{z - \frac{13}{300}c^2} \pm \frac{1}{2}\sqrt{-z - \frac{13}{150}c^2 + \frac{\frac{17}{300}c^3}{\sqrt{z - \frac{13}{300}c^2}}} \\ \text{or, } \sqrt[11]{abc^9} = \frac{7}{20}c \pm \frac{1}{2}\sqrt{z - \frac{13}{300}c^2} \pm \frac{1}{2}\sqrt{-z - \frac{13}{150}c^2 - \frac{\frac{17}{300}c^3}{\sqrt{z - \frac{13}{300}c^2}}} \end{array}\right\}$$

This, involved to the 11th power, will yield an equation, which shall have no other surds than quadratic and cubic; and, since these may be removed, whatever be their number, it is evident, that an equation may be at length deduced free from all surds: But the accomplishment of this would require so great labour, that it may at present suffice, to have shewn the possibility, and pointed out the method, of removing all surds from an equation consisting of 3 surds of the 11th power.

Far greater would be the labour to exterminate 3 surds of the 13th power.

Surds of the 12th power, it must already have sufficiently appeared, may be taken away in any number, according to the principles of extermination of cubic and quadratic surds.

It is also sufficiently manifest, that, if an equation, consisting of 3 surds of a certain power (*v. g.* the 7th), may be cleared of surds, an equation containing 2 such surds, together with any number of other surds whose extermination is unlimited, may be also cleared of surds; and that surds, whose extermination, as to their number, is unlimited, may be exterminated from any equation containing them, however diverse they be from each other.

Thus, has been pointed out, the extermination from equations, of surds whose indices do not exceed the number 6, or of any combinations of such surds, in any number; of three surds, whose common index is either of the prime numbers between 6 and 12, or whose indices are either of these multiplied by any numbers, or powers of any numbers under 6, provided the equation contain no other quantity; also, of two surds, whose common index is, or, whose indices are, as last described, with an indefinite number of surds of the former description.

It only remains, however, for the complete establishment of the last observation, to note, that any surds, contained in the denominator of any fractional quantity of an equation, which cannot be transferred to the numerator, by multiplying both terms by a residual, as some have recommended to be done, may, by multiplying the whole equation by that denominator, be transferred to the other quantities, or numerators, of the equation.

That observation will then hold of the surds therein named, however they be situated in the equation; whether they be in the numerators, or in the denominators of fractions.

P. S. Dr. WARING's method of taking away surds* is very ingenious. It is however evidently limited by the same postulate, which restricts the application of my general method; viz. to solve an equation of the dimension next lower than the index of the surd, being prime; for this must be effected in order to obtain the imaginary values of the surd as required by his method; and this, and sometimes less than this, is sufficient in mine.

e. g. To obtain the imaginary roots of the 5th power of unity, the biquadratic equation $a^4 + a^3 + a^2 + a + 1 = 0$ must be solved. These roots are $\frac{-1-\sqrt{5} \pm \sqrt{-10+2\sqrt{5}}}{4}$ and $\frac{-1+\sqrt{5} \pm \sqrt{-10-2\sqrt{5}}}{4}$, coefficients troublesome enough, especially from their variety, as WARING himself has observed.

* *V.* Meditat. Algebr. Ed. 3. p. 152, Prob. 26.

The imaginary values of higher surds of prime indices, when found, would be still more complicated: and it is not very easy to find, for example, the imaginary values of a surd of the 11th power.*

* The imaginary roots of the 7th power of unity are,

$$(1,2;)\ -\frac{1}{6}+\frac{1}{6}\sqrt[3]{\frac{7-21\sqrt{-3}}{2}}+\frac{1}{6}\sqrt[3]{\frac{7+21\sqrt{-3}}{2}}$$
$$\pm\frac{1}{6}\sqrt{\begin{array}{l}-21-2\sqrt[3]{\frac{7-21\sqrt{-3}}{2}}-2\sqrt[3]{\frac{7+21\sqrt{-3}}{2}}\\ -\sqrt[3]{\frac{637+147\sqrt{-3}}{2}}-\sqrt[3]{\frac{637-147\sqrt{-3}}{2}}\end{array}}$$

$$(3,4;)\ -\frac{1}{6}+\frac{1+\sqrt{-3}}{12}\sqrt[3]{\frac{-7+21\sqrt{-3}}{2}}+\frac{1-\sqrt{-3}}{12}\sqrt[3]{\frac{-7-21\sqrt{-3}}{2}}$$
$$\pm\frac{1}{6}\sqrt{\begin{array}{l}-21+1+\sqrt{-3}\sqrt[3]{\frac{7-21\sqrt{-3}}{2}}+1-\sqrt{-3}\sqrt[3]{\frac{7+21\sqrt{-3}}{2}}\\ +\frac{1-\sqrt{-3}}{2}\sqrt[3]{\frac{637+147\sqrt{-3}}{2}}+\frac{1+\sqrt{-3}}{2}\sqrt[3]{\frac{637-147\sqrt{-3}}{2}}\end{array}}$$

$$(5,6;)\ -\frac{1}{6}+\frac{1-\sqrt{-3}}{12}\sqrt[3]{\frac{-7+21\sqrt{-3}}{2}}+\frac{1+\sqrt{-3}}{12}\sqrt[3]{\frac{-7-21\sqrt{-3}}{2}}$$
$$\pm\frac{1}{6}\sqrt{\begin{array}{l}-21+1-\sqrt{-3}\sqrt[3]{\frac{7-21\sqrt{-3}}{2}}+1+\sqrt{-3}\sqrt[3]{\frac{7+21\sqrt{-3}}{2}}\\ +\frac{1+\sqrt{-3}}{2}\sqrt[3]{\frac{637+147\sqrt{-3}}{2}}+\frac{1-\sqrt{-3}}{2}\sqrt[3]{\frac{637-147\sqrt{-3}}{2}}\end{array}}$$

as may be found by solving the equation $x^6+x^5+x^4+x^3+x^2+x+1=0$.

Here, in justice to Dr. WARING, I must observe, that the application of his method to the extermination of the higher surds of prime indices, may, in all cases, be brought within the condition of solving an equation, whose dimension is half the index of the surd diminished by unity. For any equation, of an even dimension, which has the coefficients, at equal distances from the middle, equal, the signs being either alike, or, as they recede from the middle, alternately opposite and alike, may in effect be reduced to half its original dimension. In that case, half of the roots of the equation are the reciprocals, or the negatives of the reciprocals, of the other half. An equation, whose roots shall be the respective sums of these pairs, will be of half the dimension of the proposed equation. Thus, if

The preceding biquadratic equation is the difference of two squares $a^4 + a^3 + \frac{9}{4}a^2 + a + 1$, and $\frac{5}{4}a^2$: consequently, its quadratic factors will be the sum and difference of the roots of these squares, viz. $a^2 + \frac{1}{2} + \frac{1}{2}\sqrt{5}.\ a + 1 = 0$, and $a^2 + \frac{1}{2} - \frac{1}{2}\sqrt{5}.\ a +$

$$x^{10} + ax^9 + bx^8 + cx^7 + dx^6 + mx^5 + dx^4 + cx^3 + bx^2 + ax + 1 = 0;$$

by the combination of corresponding terms, which has, in the preceding sheets, been repeatedly exemplified, and by division of the roots by x, may be obtained,

$$x \pm \frac{1}{x} + a \cdot \sqrt{x \pm \frac{1}{x} \mp 5} + b \cdot \sqrt{x \pm \frac{1}{x} \mp 4a} + c \cdot \sqrt{x \pm \frac{1}{x} \mp 3b + 5} + d \cdot \sqrt{x \pm \frac{1}{x} \mp 2c + 2a} + m = 0.$$

If this equation could be solved, the root being called u, the solution of the quadratic, $x^2 - ux \pm 1 = 0$, would give the root of the proposed equation, which, when every coefficient is unity, yields, $\sqrt{x + \frac{1}{x}} + x + \frac{1}{x} - 4 \cdot \sqrt{x + \frac{1}{x}} - 3 \cdot x + \frac{1}{x} + 3 \cdot \sqrt{x + \frac{1}{x}} + 1 = 0$.

If this equation could be solved, then would Dr. Waring's method serve for the universal extermination of surds of the 11th power.

I may also observe, that my method universally holds for exterminating quadrato-cubic surds, without the solution of any higher equation than a quadratic, as may appear from a former example, though the observation was not before made; and, in like manner, that it universally holds for exterminating surds of the 7th power, within the condition of solving a cubic equation. For the resulting equation of six dimensions may be reduced to one of three, independently on the simplicity or composition of the third quantity of a given combination of surds. Thus, if $\sqrt[7]{a} + \sqrt[7]{b} + \sqrt[7]{c} = 0$, $\sqrt[7]{a^4b} + 3\sqrt[7]{a^3b^2} + 5\sqrt[7]{a^2b^3} + 5\sqrt[7]{a^3b^4} + 3\sqrt[7]{a^4b^5} + \sqrt[7]{ab^6} = \frac{-a-b-c}{7}$: then after multiplying by b^3, may be obtained this cubic equation, $\sqrt[7]{a^3b^{15}} + b^2\sqrt[7]{ab^{13}} + 2b^3 \sqrt[7]{a^2b^{12}} + b^2\sqrt[7]{ab^8} + b^4 \cdot \sqrt[7]{a^2b^{12}} + b^2\sqrt[7]{ab^8} = \frac{-a-b-c}{7} \cdot b^3$. Here, whatever may be the value of c, a and b may be freed from the radical sign of the 7th power, by the solution, first of a cubic, then of a quadratic, equation, and afterwards involution.

So that both Waring's method and mine universally hold, until we arrive at surds of the 11th power; and according to mine, three such may be exterminated.

$1 = o$. These quadratic equations being solved, will give the four roots of the biquadratic, as above.

So may be solved any biquadratic equation, the coefficient of whose 4th term is equal to the product of the coefficient of the 2d by the square root of the last, the coefficient of the 1st term being unity: and, by the intervention of a cubic equation, any biquadratic may be so transformed.

III. *Analysis of a new Species of Copper Ore.* By Thomas Thomson, *M. D. F. R. S. L. and E.*

Read November 18, 1813.

The mineral which constitutes the subject of this paper was discovered by Dr. Benjamin Heyne about the year 1800 in the peninsula of Indostan, near the eastern border of the Mysore. From Dr. Heyne's description, it is probable that it occurs in nests in primitive rocks, which seem to be green stone, or at least connected with primitive trap. These rocks appear to be subordinate to mica slate. But I purposely omit all particular details, because I understand Dr. Heyne has himself a work in the press, in which the mineralogy of this country will be particularly described.

Copper mines had been wrought in these mountains some centuries ago; but they had been abandoned probably on account of the various revolutions to which this part of India has been subjected. The most common ore which occurs in these mountains is malachite, and it seems to occupy very extensive veins; but the species which I propose to describe here, occurs also in considerable quantity. It had been already made the subject of various experiments, with a view to determine how much copper it contained, but I am not aware that any person had subjected it to a regular chemical analysis, or recognised it as a new species.

All the specimens of this ore which I have seen are amorphous; so that, as far as is known at present, it never occurs

crystallized. Quartz crystals indeed are imbedded in it abundantly and very irregularly. Sometimes they are single, sometimes they constitute the lining of small cavities to be found in it. These crystals are all translucent. In some rare cases they are colourless; but by far the greater number of them are tinged of a yellowish red, and some few of them are green. The mineral is likewise interspersed with small specks of malachite; and with dark, brownish red, soft, particles, which I found to consist of red oxide of iron.

The colour varies in consequence of the irregular distribution of these extraneous substances. One specimen, which was the most free from the malachite and the red particles, was of a dark blackish brown colour. But in general the colour is a mixture of green, red, and brown; sometimes one, and sometimes another prevailing. Small green veins of malachite likewise traverse it in different directions.

The fracture is small conchoidal, and in some parts of the mineral there is a tendency to a foliated fracture. The lustre is glimmering, owing, I conceive, to the minute quartz crystals scattered through it. The kind of lustre is resinous; and on that account and the variety of colours, this ore has a good deal of the aspect of serpentine.

It is soft, being easily scratched by the knife. It is sectile. The streak reddish brown. The specific gravity 2,690.

It effervesces in acids and dissolves, letting fall a red powder. The solution is green, or blue, according to the acid, indicating that it consists chiefly of copper.

After a few preliminary trials to ascertain the nature of the constituents of this ore, I adopted the following mode of analysis.

1. 100 grains in the state of a coarse powder were put into a phial containing diluted sulphuric acid, and the mouth of the phial was stopped with cotton wool. The loss of weight, when the effervescence was at an end, amounted to 16.7 grains. This loss was owing to the escape of carbonic acid gas.

2. 100 grains of the ore were treated in the same way with muriatic acid. The green solution was decanted off, and evaporated nearly to dryness, to get rid of the excess of acid. A plate of zinc was then put into the liquid previously diluted with water. The copper precipitated weighed 48.5 grains.

On repeating the analysis, I found that the muriatic acid had likewise taken up a portion of iron. I therefore supersaturated the solution with ammonia, and threw the whole upon a filter. By this means the red oxide of iron was separated. The ammoniacal solution was then neutralized by muriatic acid, and the copper thrown down by a plate of zinc. But during my first analysis, none of the iron was taken up by the cold muriatic acid, owing, no doubt, to the state of its aggregation.

3. The red powder, which remained undissolved after the muriatic solution was drawn off, was boiled for several hours in nitromuriatic acid. The matter gradually diminished in bulk and became white, while the acid acquired a golden yellow colour. The acid was now separated from the undissolved powder, evaporated nearly to dryness to get rid of the excess of acid, diluted with water, and mixed with an excess of ammonia. A brown powder fell, which was separated by the filter, and which, after being exposed to a red heat, weighed

19.5 grains. A little of this powder being dissolved in muriatic acid was precipitated dark blue by prussiate of potash. The remainder being mixed with tallow, and suddenly heated in a covered crucible, became black, and was attracted by the magnet. These properties leave no doubt that the powder was peroxide of iron.

4. The ammoniacal solution had a light blue colour, I therefore neutralized it by muriatic acid, and put into it a polished plate of zinc. I obtained a sensible deposit of copper; but so small, that I was unable to collect and weigh it. I estimate it at about 0.1 grain.

5. The white undissolved matter being heated to redness weighed 2.1 grains. On examining this matter attentively, I found it entirely composed of fragments of quartz crystals which had been interspersed through the ore, and had from their minuteness escaped my observation.

6. From the facility with which the copper dissolved in muriatic and sulphuric acids, there could be no doubt that it existed in the ore in the state of an oxide. But the red colour of the ore made me uncertain whether the oxide was the red or the black. I therefore put 100 grains of the ore into a tall narrow phial, filled the phial full of water, and then by means of a funnel poured a quantity of muriatic acid into the bottom of the vessel. The ore was immediately attacked, and the solution from the very commencement appeared green. This I consider as a demonstration that the copper in the ore was in the state of black oxide. Now, black oxide of copper is a compound of 100 metal + 25 oxygen. So that the 48.6 grains of copper, extracted from the ore when in the state of black oxide, must have weighed 60.75 grains.

From the preceding analysis, it appears that the ore is composed as follows:

Carbonic acid - -	16.70
Peroxide of copper	60.75
Peroxide of iron -	19.50
Silica - -	2.10
Loss - -	0.95
	100.00

The silica was obviously accidental, and derived from the quartz crystals; so that the ore, in fact, consists of carbonic acid, peroxide of copper, and red oxide of iron. In the different analyses, I have found the copper to vary a little. The least quantity I obtained was 48.6, the greatest 51 grains. The red oxide of iron and siliceous matter varied also somewhat. The least quantity obtained was 19 grains, the greatest 25 grains. These variations are owing chiefly to the admixture of quartz crystals, and partly to the specks of malachite and red oxide of iron with which the ore is interspersed.

The carbonic acid is obviously combined with the black oxide of copper, so as to constitute carbonate of copper. Now carbonate of copper, as I ascertained by a direct analysis, is composed of an integrant particle of carbonic acid, and an integrant particle of black oxide of copper. An integrant particle of carbonic acid, as I have shewn elsewhere, weighs 2.751, and an integrant particle of peroxide of copper weighs 10. Now 2.751 is to 10 as 16.7 is to 60.75, so that there can be no doubt that the carbonic acid and oxide of copper are united in the ore. As to the oxide of iron, I am disposed to consider it as only mechanically mixed; because in one expe-

riment I dissolved almost all the copper without touching the iron. Yet it deserves attention, that 77.4 and 19.5, the weight of carbonate of copper and oxide of iron found by the preceding analysis, correspond with three integrant particles of carbonate of copper, and one integrant particle of peroxide of iron.

We were previously acquainted with two other native species of this salt, namely, *malachite* and *blue carbonate* or *copper azure*. But both of these are hydrous carbonates containing water as a constituent, and if any confidence be put in the analyses of KLAPROTH, whose precision is sufficiently known, malachite contains twice as much water as the blue carbonate. Blue carbonate is a compound of one integrant particle of water, and one integrant particle of carbonate of copper, while malachite contains two particles of water. Our ore is an *anhydrous carbonate of copper*. When heated to redness, it loses its carbonic acid, but undergoes no further change. Some specimens lost about half a grain more than their carbonic acid. This I ascribed to the water in the malachite, with which the ore was occasionally mixed.

IV. *The Bakerian Lecture: on some new Electro-chemical Phenomena.* By William Thomas Brande, *Esq. F. R. S. Prof. Chem. R. I.*

Read November 25, 1813.

§. I.

It has been ascertained by Sir H. Davy, that when compound bodies capable of transmitting electricity are submitted to the operation of the Voltaic pile, their proximate and ultimate elements are separated with uniform phenomena; that acids are attracted towards the positively electrified surface, and that alkaline and inflammable substances take an opposite direction and collect at the negative pole.

Of the *ultimate* chemical elements of bodies, the greater number exhibit the last mentioned character, and a few only appear to be attracted towards the positive extremity of the Voltaic instrument; and as bodies possessed of dissimilar electrical powers *attract* each other, it has been concluded, that the inherent electrical state of the former is positive, that of the latter negative.

These chemical effects were at first regarded as peculiar to the Voltaic pile, and were considered to depend upon the operation of a new agent, termed the Galvanic fluid, until Dr. Wollaston, in the year 1801,* succeeded in imitating the effects by means of the electrical machine, and thus

* Phil. Trans. 1801, p. 435.

experimentally demonstrated the identity of common and Voltaic electricity.

More recent investigations, and especially the admirable researches of Sir H. DAVY, have fully established the correctness of these views, and have shewn that the different action of the Voltaic pile and the electrical machine, depends chiefly upon the *quantity* of electricity in the former being great, while its *intensity* is inconsiderable, and *vice versâ*.

In the following Lecture I shall have the honour of presenting the Royal Society with some new inquiries connected with these objects of research, and have much pleasure in adducing facts which throw further light upon this interesting department of chemical science, and which harmonize with the opinions of the able philosophers alluded to.

§. II.

When the flame of a candle is placed between two surfaces in opposite electrical states, the negative surface becomes most heated: this circumstance was considered by Mr. CUTHBERTSON as indicating the passage of electric fluid from the positive to the negative surface.*

Mr. ERMAN † has shewn that certain substances are *unipolar* in regard to the electricity of the Voltaic pile; that is, that they are only susceptible of transmitting one kind of electricity. The insulated flames of wax, of oil, of spirit of wine, and of hydrogen gas, only conduct positive electricity; dry soap, on the contrary, and the flame of phosphorus, under the same circumstances, only transmit negative electricity.

* Practical Electricity.

† Annales de Chimie, 1807, Tom. LXI. p. 113.

Sir H. DAVY* considers the result of Mr. CUTHBERTSON's experiment to depend upon the *unipolarity* of the flame, which would render it positive, and cause it to be attracted towards the negative pole.

On perusing these statements, it occurred to me that they admitted of another explanation, and that the appearances might be connected with the chemical nature of the substances employed. I repeated Mr. CUTHBERTSON's experiment, and found that when the electrical machine was in weak action, the negative surface not only became hot sooner than the positive, but that the smoke and flame of the candle were visibly attracted towards it. I now removed the candle, and substituted the flame of phosphorus, when the appearances were exactly reversed: the positive surface now became considerably warmer than the negative, and the flame and smoke of the phosphorus were powerfully directed upon it. I conceived, therefore, that the flame of the candle was attracted by the negative pole, in consequence of the carbon and hydrogen in which it abounds, and that the rapid formation of acid matter during the combustion of the phosphorus, was the cause of its attraction towards the positive pole: in short, that the appearances were consistent with the known laws of electro-chemical attraction.

To ascertain the correctness of this idea, it became necessary to examine the phenomena with greater precision, and to institute the more extended series of experiments, which, with their results, I shall proceed to detail.

The apparatus employed consisted of two insulated brass balls capable of being brought near to, or removed from each

* Elements of Chem. Philos. Vol. I. p. 177.

other, with a small table between them, as represented in the annexed drawing. (Pl. II.)

When it was necessary to ascertain the relative temperatures of the balls with accuracy, I made use of the instrument, *fig.* 2, devised by Mr. PEPYS, in which *a a* represent two thin spheres of brass containing delicate thermometers, the bulbs of which, as well as the interior of the spheres, were coated with lamp black, to render the increase of temperature more evident. One of the balls was connected with the negative, the other with the positive conductor of a small electrical machine of Mr. NAIRNE's construction, as represented in fig. 1, so that the apparatus was perfectly insulated.

§. III.

Exp. 1. A small stream of olefiant gas was burned between the balls. The flame was evidently attracted towards the negative side. The combustion was continued for one minute. At the commencement of the experiment, the mercury in the thermometers stood at 60° FAHRENHEIT; at its termination the positive thermometer indicated 62°, the negative 72°.

2. The experiment was repeated with sulphuretted hydrogen: the flame was now only slightly attracted by the negative ball, but the sulphureous acid vapour passed off towards the positive surface.

In this experiment the positive thermometer rose 3°, and the negative thermometer 6° in one minute.

3. A very small flame of phosphuretted hydrogen was slightly bent towards the positive pole. In one minute it produced an effect upon the positive thermometer = 5°, upon the negative = 3°. When a larger flame was used, it

appeared equally drawn towards the two electrical poles, but the acid vapour passed off in the direction of the positive ball.

4. The flame of arsenicated hydrogen was attracted by the negative surface: the fumes of white arsenic produced during the combustion were slightly attracted towards the positive pole.

5. The flame of hydrogen appeared weakly attracted by the negatively electrified ball, but on employing the apparatus with thermometers, the rise was nearly equal on both sides. In one experiment, made with much caution, the thermometers stood at its commencement at 56°. The gas was burned for two minutes: the negative thermometer rose to 62°, the positive to 61°.

In a second experiment, the combustion was continued for five minutes: the effect upon the negative thermometer was = 4°, that upon the positive = 2,5°.

6. With a very small stream of carbonic oxide, the results were still less distinct. The tip of the flame appeared in some cases to be slightly inclined towards the positive ball, but one thermometer was not more affected than the other.

On substituting a larger current of the gas, which produced a flame between three and four inches in length, it was much more evidently directed to the positive surface, but the rise in the positive thermometer was less than might have been expected. In several cases where the combustion was continued for two minutes, the rise in the positive thermometer did not exceed that of the negative more than 2,5 to 3 degrees.

7. Sulphur, in a state of combustion, was placed between the oppositely electrified balls. The flame being extremely

small, its direction could not be observed, but the sulphureous acid vapour was attracted by the positive ball.

8. The *flame* of the sulphuret of carbon (sulphuretted liquor of LAMPADIUS) was attracted by the negative surface: the *acid fumes* which it throws off took an opposite direction.

9. When phosphorus in a state of vivid combustion was introduced into the electrical circuit, the flame, and the phosphoric acid produced, were powerfully directed towards the positive surface.

The acid vapour which arises from phosphorus, during its slow combustion in a warm atmosphere, is also attracted by the positively electrified ball.

10. A small stream of muriatic acid gas was allowed to pass into the atmosphere between the electrified spheres; it was immediately attracted to the positive pole. This effect is rendered more evident by the diffusion of a small quantity of ammonia through the atmosphere of the room in which the experiment is made.

The attraction of the muriatic acid is strikingly exhibited by coating the conductors with litmus paper, placing them at a distance of about six inches asunder, and propelling a current of the gas through a small tube between them. If the electrical machine is not in too powerful action, the positive ball is instantly reddened, while the blue colour of the negative surface remains unaltered.

11. Nitrous acid gas exhibited the same appearances as muriatic acid.

12. Potassium in a state of combustion was placed between the electrified surfaces. Its flame, and the alkaline fumes it produces, were drawn to the negative conductor.

13. Ammonia afforded no very distinct results. I attempted to ascertain its electrical state by disengaging it through a small tube placed between the conductors, the atmosphere being slightly tainted with muriatic acid; but it was apparently equally attracted and repelled by the electrical surfaces. When the conductors were coated with turmeric paper, the negative appeared sooner reddened than the positive, but in a very short time the effect upon each became equal.

14. Benzoic acid evolved by gently heating benzoin between the electrical poles, was attracted to the positive side; but when the balsam took fire, the flame and carbonaceous matter were immediately drawn in an opposite direction.

I heated some pure benzoic acid placed upon a silver capsule, between the balls; as long as the temperature was only sufficient to raise it in vapour, it took the positive direction; but the moment that the acid was inflamed, the carbonaceous fumes passed to the negative side.

15. Camphor during its combustion throws off a large quantity of charcoal, and when burned between the opposite electrical surfaces, furnishes a good instance of the attraction of that substance by the negative pole, which soon becomes coated with it, the positive ball being much less soiled.

16. The resinous bodies in general exhibited the same appearances as camphor: when in a state of combustion, both the flame and smoke were repelled by the positive, and attracted by the negative pole.

17. Amber, on the contrary, presented phenomena nearly similar to those of benzoin. When brought into a state of fusion, its acid vapour was attracted by the positive ball, but when made to burn, an opposite effect was produced.

§. IV.

In the preceding detail of experiments, such only have been selected as were attended with marked results. The electrical energies of many other substances were examined and compared, but no new facts were ascertained, nor were any circumstances observed which interfere with the inferences suggested by the above statement.

To insure accuracy, each of the experiments was several times repeated in the presence and with the assistance of some of my chemical friends, and the results were uniform when proper precautions were attended to. It is especially necessary to operate in a quiet atmosphere, and to employ a feeble electrical power, for if the machine is too strongly excited, the substances under examination are alternately attracted and repelled between the poles, especially if insulated, or non-conductors of electricity. The balls were usually withdrawn from each other to a distance of four inches, and the subject of experiment placed equidistant between them. The poles too were occasionally changed, by moving the connecting wires from one ball to the other, with a view to insure correctness by observing the change thus produced in the direction of the flame or vapour.

The experiments were sometimes varied, by using the Leyden jar with a sliding electrometer connected with its outer surface: the substance, the electricity of which was to be examined, was placed between the ball of the charged jar, and that of the electrometer.

Regarding these experiments, as connected with electro-chemical theory, they appear to furnish a more evident proof

than has hitherto been offered, of the inherent electrical states of matter, which are decidedly exhibited by the attractions and repulsions between the opposite poles; and when connected with Dr. WOLLASTON's researches, to which I have elsewhere alluded, they amply demonstrate the identity in chemical powers of common and Voltaic electricity.

The attraction of acids by the positively electrified surface, and of alkalies and inflammables by that which is negatively electrified, is thus easily exhibited; and the theory which regards their mutual attractive energies, as dependent upon their opposite electrical states, confirmed by experiments, not less decisive, than those in which the Voltaic instrument was employed.

Of the former class phosphorus in slow, and in rapid combustion, and benzoic acid, furnish the most striking instances; and of the latter, the combustion of potassium and of camphor are excellent examples.

There are, however, some circumstances which appear difficult to explain, and which have not turned out as might have been expected *a priori*. The combustion, for instance, of carburetted hydrogen gives rise to the production of water and carbonic acid, but its flame is entirely and powerfully attracted by the negative surface; and carbonic oxide, which produces carbonic acid, is not very evidently attracted by either pole, unless the flame be of a very large size. It may, however, be conceived with regard to carburetted hydrogen, that the carbonic acid which is formed does actually pass off to the positive surface, and that the polarity of its flame is influenced rather by the combustible, than by the product of combustion: this idea is in some measure sanctioned by the appearances

exhibited by the flame of the sulphuret of carbon, which is directed towards the negative ball, although the sulphureous acid visibly passes towards the positive side, and it is fair to infer, that the carbonic acid accompanies it.

An attempt was made to detect the carbonic acid by means of caustic potash: for this purpose a piece of linen, moistened with the alkaline solution, was applied to each conductor, and a current of carbonic acid, issuing from a small tube, was directed between them. The linen was then put into dilute muriatic acid, and it was expected that the effervescence would be greatest in that removed from the positive pole; this generally appeared to be the case, but I cannot say that the results were satisfactorily distinct, nor indeed does the method admit of the accuracy required.

The experiments related in the second section of this lecture, suggest an explanation of the phenomena alluded to, as observed by Mr. ERMAN, more consonant with the known laws of electricity than that which he has suggested. The flame, for instance, of oil, wax, &c. must be considered as consisting chiefly of those bodies in a state of vapour, and their natural electricities being positive, it is obvious, that, when connected with the *positive* pole of the battery, and with a gold leaf electrometer, the leaves will continue to diverge, but when applied to the *negative* pole, that electrical state will be annihilated by the inherent positive energy of the flame, and consequently the leaves of the negative electrometer will not diverge. On the other hand, the flame of phosphorus is negatively unipolar. Now it has been shewn, that this flame, (owing probably to the rapidity with which it is forming a powerful acid by combination with a large portion of oxygen,)

Fig. 1.

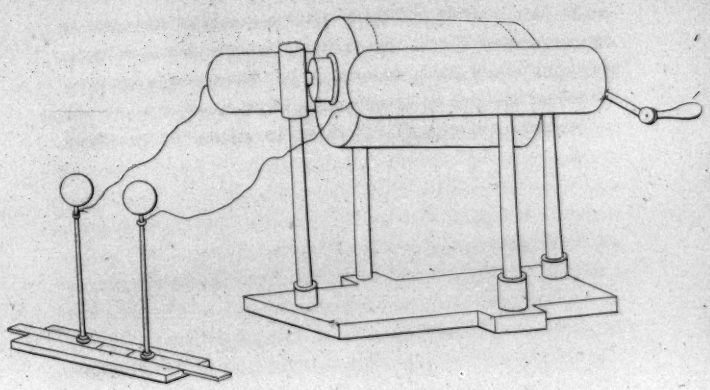
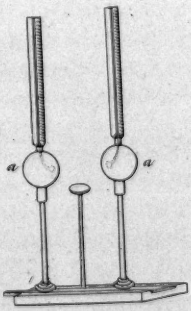


Fig. 2.



is attracted by the positively electrified surface, and consequently that it is itself negative, so that it would transmit negative electricity to the electrometer, but would annihilate the positive power, and thus appear as an insulator under the particular circumstances which Mr. ERMAN has described.

V. *An Account of some new Experiments on the fluoric Compounds; with some Observations on other Objects of Chemical Inquiry.* By *Sir* H. Davy, *LL.D. F.R.S. V.P.R.I.*

Read February 13, 1814.

In this Paper I shall offer to the Society a continuation of those researches, the details of which have been already honoured with a place in their Transactions; and I trust, that the experiments and observations which I have to communicate, will be found to elucidate some important but obscure parts of chemical philosophy.

In the last Paper, which I had the honour of presenting to this Body, I have given an account of a number of experiments made with a view of decomposing the fluoric acid: the most probable inference, from my results, was that the pure liquid fluoric acid consists of hydrogen united to a substance, which, from its strong powers of combination, has not as yet been procured in a separate form, but which is detached from hydrogen by metals, and which, in union with the basis of the boracic acid and silica, forms the fluo-boric and silicated fluoric gases.

All the new experiments, that I have made on the fluoric compounds, tend to confirm this idea; and the various attempts that I have made, since the last session, to decompose the principle in the fluoric acid separated at the negative surface in Voltaic combinations, have been unsuccessful.

I have found that fluate of lead, the substance formed by

the action of a soluble salt of lead on fluate of potassa or fluate of ammonia, is immediately decomposed, when moist, by ammoniacal gas, and a white powder separates from it, which must contain oxygen, as it gives carbonic acid by being ignited with charcoal; but dry fluate of lead may be fused in ammonia without undergoing the slightest alteration, and the presence of water, which may furnish oxygen to the lead and hydrogen to the fluoric principle, seems absolutely necessary for the production of oxidated bodies from the fluoric combinations.

I obtained analogous results by acting on silicated fluate of ammonia and fluo-borate of ammonia by chlorine; when the salts were moist, or when the gas was not free from vapour, silica and boracic acid were formed in small quantities, but when water was carefully excluded, these bodies did not appear; and the results were muriate of ammonia and silicated fluoric gas, or fluo-boric gas and azote.

I ignited two points of charcoal intensely in fluo-boric and silicated fluoric acid gases, pure, and mixed with oxygen; but no change, indicating a decomposition, took place; the only new product was a little inflammable gas, which was probably disengaged from the charcoal.

I passed pure liquid fluoric acid over charcoal ignited to whiteness, in a platinum tube; no carbonic acid was formed, and a very minute quantity of gas only was produced, which proved to be hydrogen.

I mentioned in my last communication to the Society, that I had made several experiments on the composition of the fluates; since that time I have repeated some of the processes, and I shall detail such of the results as appear to be most

correct. In experiments on the fluates, it is very difficult to exclude sources of inaccuracy; glass vessels cannot be used, and even silver and platinum vessels are slightly acted upon by the pure fluoric combinations soluble in water.

Fluor spar decomposed by sulphuric acid increases more than three-fourths of its original weight; but to produce this result several successive distillations of it with fresh portions of acid are necessary, and the spar must be in very fine powder, and the result must be powdered after every operation.

In an experiment made in a crucible of platinum, in which very pure white Derbyshire spar was used, and sulphuric acid distilled in glass vessels, and in which the product was heated to whiteness in every operation, 100 grains became in the first operation 159.4 grains, which gained

in the second	6.9
in the third	4.2
in the fourth	1.9
in the fifth	1.6
in the sixth	0.9
in the seventh	0.8
in the eighth	-.0
Total increase	75.2

If this result be calculated upon, supposing the number representing calcium to be 40, as I have given it in my Elements of Chemical Philosophy, the number representing fluorine will be 34.2, and fluor spar must be supposed to be composed of 40 calcium, and 34.2 fluorine.

22 grains of fused subcarbonate of potassa decomposed by diluted liquid fluoric acid, in an experiment made with great care, was found to afford 18.15 grains of dry fluate of potassa;

and by a very careful analysis, it was found that this subcarbonate contained 31 per cent. of carbonic acid; now, if the remainder of the salt be supposed to be pure potassa, and the calculations be made on this idea, it will appear that fluate of potassa must consist of 125.9 of potassium and 54.74 of fluorine, and the number representing fluorine must be regarded as 32.6.

The 18.15 grains of fluate of potassa decomposed by sulphuric acid afforded 38.5 of acid sulphate of potash. Supposing acid sulphate of potassa to consist of two proportions of sulphuric acid 150, and one of potassa 90, they ought to have given 40 grains, and this loss is no more than might be expected in the process of decomposition and evaporation.

In some experiments that I made on the decomposition of the hydrates of potassa and soda, when decomposed by fluoric acid, the results indicated a number for fluorine a little lower. Thus 20 grains of hydrate of potassa were converted into 19.8 grains of fluate of potassa, and 20 grains of hydrate of soda into 19.6 of fluate of soda; but I do not place so much confidence in these results, as there always was great heat produced during the action of the acid upon the hydrates; and probably a minute quantity of the hydrates might have been dissipated at the beginning of the process.

It appears reasonable to conclude, as I have stated in my last paper, published in the Philosophical Transactions, that the number representing fluorine is less than half of that representing chlorine, about 33.

I endeavoured to ascertain the composition of fluate of ammonia, by adding together hydro-fluoric acid and solution of ammonia of known composition, and I found in this way, that

100 parts of solution of ammonia of specific gravity 9162 required for its saturation 52 grains of diluted fluoric acid of such a strength, that an equal portion produced exactly 32 grains of fluate of potassa. According to this experiment fluate of ammonia must consist of 9.7 of fluorine to 22 of ammonia;* and, if it be conceived that liquid fluoric acid consists of two proportions of hydrogen to one of fluorine, the true composition of fluate of ammonia will be one proportion of fluoric acid 35, and two proportions of ammonia 64.†

The volatility of fluate of ammonia rendered it impossible to ascertain by evaporation the real quantity of solid salt formed, though the heat was never raised so high as that of boiling water, yet only 12.7 grains of solid fluate of ammonia could be procured.

Two hundred cubical inches of ammonia, which weigh 36 grains, condense one hundred cubical inches of silicated fluoric gas, weighing, under the same circumstances, 110.7, and if it be supposed that silicated fluate of ammonia contains one proportion of silicated fluoric acid and one of ammonia, then the number representing silicated fluoric gas will be 98.4, and it may be conceived to consist of two proportions of fluorine 66, and one of the siliceous basis 32.4.

According to the experiments of JOHN DAVY, 100 parts of silicated fluoric acid afford 61.4 of silica when decomposed by aqueous solution of ammonia, hence silica may be conceived to consist of 32.4 of basis and 29 of oxygen, and this is very near two proportions of oxygen.

* See Elements of Chemical Philosophy, page 268.

† That is, if ammonia be regarded as composed of one proportion of azote 26, and six of hydrogen.

I decomposed 20 grains of silicated fluate of ammonia by solution of ammonia, and I obtained 9.2 grains of silica. This result offers proportions very little different from those gained in the preceding calculations.

I have made some direct experiments with the hopes of determining the proportions of oxygen in silica, but they have been unsuccessful; I have ascertained, however, that it requires more than three parts of potassium to decompose one part of silica, which shews that this substance cannot contain much less than half its weight of oxygen.

I have endeavoured to separate the siliceous basis in a pure form, with the view of making synthetical experiments on its nature by combustion in oxygen, and my results, though not perfectly satisfactory, yet seem worthy of notice, and may lead to more successful attempts.

I decomposed silica by passing potassium in excess through it, in a heated tube of platinum: the result consisted chiefly of alkali containing a dark coloured powder, the basis of silica diffused through it. I fused the whole mass with sulphur, which, in combining with the dry alkali, produced ignition. I attempted to detach the sulphuret of potassa by water: in this case the dark particles separated, but during their separation and after, they acted upon the water of the solution producing gas, and, in attempting to collect them by the filter, I failed to procure sufficient for examination, for they were principally converted into silica.

I heated the substance procured in another experiment of this kind with hydrate of potassa; in this case there was a copious effervescence, and silica appeared to be reproduced and dissolved by the alkali.

I heated a portion of a similar result in strong lixivium of potassa; the solution gained a tint of olive, but there was scarcely any effervescence; from this it seems probable, that the inflammable basis of silica, like boron, is soluble in alkaline solutions without decomposing them.

Indeed this body, in its general characters, appears very analogous to boron. It appears to be neither volatile nor fusible; its oxide exerts, like boracic acid, a neutralizing power on the alkalies, though of a feebler kind, and forms, like boracic acid, vitreous bodies with the alkaline earths, and, like boron, the siliceous basis in combination with fluorine constitutes a powerful acid.

In my first views of the nature of the boracic and siliceous bases, I thought it probable that they would both appear as metals, if they could be entirely freed from oxygen; but it now seems more probable, that they form a class by themselves, offering a kind of link in the chain of natural bodies, when arranged according to their analogies, between charcoal, and sulphur and phosphorus.

It seems worthy of an experimental inquiry, whether the siliceous basis may not be obtained pure by heating the result procured from silica by potassium with pure sulphuric acid, which might possibly detach the potassa to form acid sulphate of potassa, without being decomposed by the inflammable basis.

I have made many new experiments with the hope of decomposing chlorine, but they have been all unavailing; nor have I been able to gain the slightest evidence of the existence of that oxygen which many persons still assert to be one of its elements.

I kept sulphuret of lead for some time in fusion in chlorine, the results were sulphurane (Dr. THOMSON's liquor) and plumbane (muriate of lead); not an atom of sulphate of lead was formed in the experiment, though if any oxygen had been present, this substance might have been expected to have been produced.

I heated plumbane (muriate of lead) in sulphurous acid gas, and likewise in carbonic acid gas, but no change was produced; now, if oxygen had existed either in chlorine, or in its combination with lead, there is every reason to believe, that the attractions of the substances concerned in these experiments would have been such as to have produced the insoluble and fixed salts of lead, the sulphate in the first case, and the carbonate in the second.

I shall not enter into any discussion upon the experiments in which water is said to be produced by the action of muriatic gas on ammonia: there is, I believe, no enlightened and candid person, who has witnessed the results of processes in which large quantities of muriate of ammonia, made by the combination of the gases in close vessels, have been distilled, without being satisfied, that there is no more moisture present, than the minute quantity which is known to exist in the compound vapours diffused through ammoniacal and muriatic acid gases, which cannot be considered either as essential to the existence of the gases, or as chemically combined with them.*

* Dr. HENRY found it very difficult to free ammonia from the aqueous vapour existing in it by hydrate of potassa, and probably the hydrated muriatic vapour which I have detected in muriatic acid gas, by a freezing mixture, is not decomposable by muriate of lime.

One of the first experiments that I made, with the hope of detecting oxygen in chlorine, was by acting upon it by ammonia, when I found that no water was formed, and that the results were merely muriate of ammonia and azote;* and the driest muriate of ammonia, I find, when heated with potassium, converts it into muriate of potassa, which result would be impossible on the hypothesis of oxymuriatic gas being a compound of oxygen, for, if there was a separation of water during the formation of the muriate, the same oxygen could not be supposed to be detached in water, and yet likewise to remain so as to form part of a neutral salt.

If water had been really formed during the action of chlorine on ammonia, the result would have been a most important one: it would have proved either that chlorine or azote was a compound, and contained oxygen, or that both contained this substance; but it would not have proved the existence of oxygen in chlorine, till it had been shewn that the azote of the ammonia was unchanged in the operation.

Some authors continue to write and speak with scepticism on the subject, and demand stronger evidence of chlorine being undecompounded. These evidences it is impossible to give. It has resisted all attempts at decomposition. In this respect, it agrees with gold, and silver, and hydrogen, and oxygen. Persons may doubt, whether these are elementary bodies; but it is not philosophical to doubt, whether they have not been resolved into other forms of matter.

By the same mode of reasoning, as that in which oxygen is conceived to exist in chlorine, any other species of matter might be supposed to form one of its constituent parts; and

* Philosophical Transactions for 1810.

by multiplying words all the phenomena might be satisfactorily explained. Thus in the simple view of the formation of muriatic acid, it is said one volume of chlorine combines with one of hydrogen, and they form two volumes of muriatic acid gas. In the hypothesis of chlorine containing oxygen, it is said, the oxygen of the chlorine combines with the hydrogen to form water, and this water unites to an unknown something, or dry muriatic acid, to produce a gaseous body. If it were asserted that chlorine contained azote, oxygen, and this unknown body, then it might be said, that, in the action of hydrogen on chlorine, the azote, the oxygen and the chlorine, having all attractions for hydrogen, enter into union with it, and form a quadruple compound.

Professor Berzelius has lately adduced some arguments, which he conceives are in favour of chlorine being a compound of oxygen from the laws of definite proportions; but I cannot regard these arguments of my learned and ingenious friend as possessing any weight. By transferring the definite proportions of oxygen to the metals, which he has given to chlorine, the explanation becomes a simple expression of facts; and there is no general canon with respect to the multiples of the proportions in which different bodies combine. Thus azote follows peculiar laws in combining with every different body; it combines with three volumes of hydrogen, with half a volume of oxygen, with 1.2 and $1\frac{1}{2}$ of the same body, and with four volumes of chlorine.

The chemists in the middle of the last century had an idea, that all inflammable bodies contained phlogiston or hydrogen. It was the glory of Lavoisier to lay the foundations for a sound logic in chemistry, by shewing that the existence of

this principle, or of other principles, should not be assumed where they could not be detected.

In all cases, in which bodies support combustion or form acids, oxygen has been supposed by the greater number of modern chemists to be present; but as there are many distinct species of inflammable bodies, so there may be many distinct species of matter which combine with them with so much energy, as to produce heat and light; and various bodies appear capable of forming acids; thus hydrogen enters into the composition of nearly as many acids as oxygen, and three bodies, namely, sulphuretted hydrogen, muriatic acid, and fluoric acid which contain hydrogen, are not known to contain oxygen. The existence of oxygen in the atmosphere, and its action in the economy of nature, and in the processes of the arts, have necessarily caused it to occupy a great portion of the attention of chemists, and, being of such importance, and in constant operation, it is not extraordinary, that a greater number of phenomena should be attributed to it, than it really produces.

In the views that I have ventured to develope, neither oxygen, chlorine, or fluorine, are asserted to be elements; it is only asserted, that, as yet, they have not been decomposed.

As the investigation of nature proceeds, it is not improbable, that other more subtile bodies belonging to this class will be discovered, and perhaps some of the characteristic differences of those substances, which apparently give the same products by analysis, may depend upon this circumstance.

The conjecture appears worth hazarding, whether the carbonaceous matter in the diamond may not be united to an extremely light and subtile principle of this kind, which has

hitherto escaped detection, but which may be expelled, or newly combined, during its combustion in oxygen. That some chemical difference must exist between the hardest and most beautiful of the gems and charcoal, between a non-conductor and a conductor of electricity, it is scarcely possible, notwithstanding the elaborate experiments that have been made on the subject, to doubt: and it seems reasonable to expect, that a very refined or perfect chemistry will confirm the analogies of nature, and shew that bodies cannot be exactly the same in composition or chemical nature, and yet totally different in all their physical properties.

VI. *Some Experiments and Observations on a new Substance which becomes a violet coloured Gas by Heat.* By Sir Humphry Davy, *Knt. LL. D. F. R. S.*

Read January 20, 1814.

A NEW and a very curious substance has recently occupied the attention of chemists at Paris.

This substance was accidentally discovered about two years ago by M. COURTOIS, a manufacturer of saltpetre at Paris. In his processes for procuring soda from the ashes of sea weeds, (cendres de vareck) he found the metallic vessels much corroded; and in searching for the cause of this effect, he made the discovery. The substance is procured from the ashes, after the extraction of the carbonate of soda, with great facility, and merely by the action of sulphuric acid:—when the acid is concentrated, so as to produce much heat, the substance appears as a vapour of a beautiful violet colour, which condenses in crystals having the colour and the lustre of plumbago.

M. COURTOIS soon after he had discovered it, gave specimens of it to M. M. DESORMES and CLEMENT for chemical examination; and those gentlemen read a short memoir upon it, at a meeting of the Imperial Institute of France, on Nov. 29th. In this memoir, these able chemists have described its principal properties; they mentioned that its specific gravity was about four times that of water, that it becomes a violet coloured gas at a temperature below that of boiling water, that it combines

with the metals and with phosphorus and sulphur, and likewise with the alkalies and metallic oxides, that it forms a detonating compound with ammonia, that it is soluble in alcohol, and still more soluble in ether; and that by its action upon phosphorus and upon hydrogen, a substance having the characters of muriatic acid is formed. In this communication they offered no decided opinion respecting its nature.

M. AMPERE had the goodness to give me some of this substance, and M. CLEMENT having requested me to submit it to some analytical tests, I made several experiments upon it, which convinced me that it was a new substance undecompounded in any of the circumstances to which I was able to expose it; and that the acid formed in processes upon it was not muriatic acid, but a new acid possessing a striking resemblance to that body.

M. GAY LUSSAC (to whom M. CLEMENT had furnished some of the substance, and with whom he had made some experiments upon it before the communication of his memoir) on Monday, Dec. 6, read to the Institute a paper, in which he stated that the acid formed by its action on hydrogen is a peculiar one. He mentioned several interesting particulars respecting the mode of its production, and he compared it to oxymuriatic gas or chlorine, and stated that two hypotheses might be formed on its nature; and that it might be considered as an undecompounded substance, or as a compound of oxygen; M. GAY LUSSAC is still engaged in experiments on this subject, and from his activity and great sagacity, a complete chemical history of it may be anticipated. But as the mode of procuring the substance is now known to the chemical world in general, and as the combinations and agencies of it offer an

extensive field for enquiry, and will probably occupy the attention of many persons, and as the investigation of it is not pursued by the discoverer himself, nor particularly by the gentlemen to whom it was first communicated, I shall not hesitate to lay before the Royal Society an account of the investigations I have made upon it; and I do this with the less scruple, as my particular manner of viewing the phenomena has led me to some new results, which probably will not be considered by the Society as without interest in their relation to the general theory of chemistry, and in their possible application to some of the useful arts.

The first experiments that I made on this substance, were to ascertain whether (argentane) muriate of silver could be formed from its solution in water or alcohol, and for this purpose it was purified by distilling it from lime. Its solution I found, when mixed with solution of nitrate of silver, deposited a dense precipitate of a pale lemon colour; this precipitate when collected and examined, proved to be fusible at a low red heat, and then became of a red colour. When acted upon by fused hydrate of potassa, it was rapidly decomposed, and a solid substance, having all the characters of oxide of silver, was formed. The matter soluble in water separated by a filter, and, acted upon by sulphuric acid, afforded the peculiar substance.

A solution of potassa, after being boiled on the precipitate, afforded the peculiar substance, when treated by the same acid.

The precipitate was much more rapidly altered by exposure to light, than the muriate of silver, and was evidently quite a distinct body.

Conceiving from the action of potassa upon it, that it must be a compound of the peculiar substance and silver, I endeavoured to form it directly by the combination of the two bodies. I introduced some of the substance into the closed end of a small glass curved tube, and placed in the upper part of it some silver foil; I heated the foil nearly to redness, and then passed the substance over it in vapour; there was an immediate action, the silver was rapidly dissolved, and a fusible substance formed, in all its obvious sensible and chemical characters, the same as that obtained from solutions of the substance by nitrate of silver.

The modes which occurred to me, as most likely to effect its decomposition by chemical agents, were the action of the highly inflammable metals upon it which unite to oxygen and chlorine, or the action of chlorine which in general tends to the expulsion of oxygen, and to the separation of inflammable bases from that principle.

I heated some potassium in a little glass tube, and passed some of the substance in vapour over it; at the moment the vapour came in contact with the potassium, there was an inflammation, and the potassium burnt slowly with a pale blue light. There was no gas disengaged when the experiment was repeated in a mercurial apparatus.

The substance formed by the action of potassium was white, fusible at a red heat, and soluble in water. It had a peculiar acrid taste. When acted upon by sulphuric acid, it effervesced, and the peculiar substance appeared.

It was evident that in this experiment there had been no decomposition of the body; the result seemed to depend merely upon the combination of it with the potassium.

I exposed the body to the action of chlorine in a small glass tube; it absorbed the chlorine, and a substance formed which was volatile by heat, and which appeared as a yellow solid; it was soluble in water, and rendered the water of a yellowish green colour and strongly acid, the solution when acted upon by solution of potassa not in excess effervesced, and afforded the peculiar substance.

The acid formed by the solution of the substance united to chlorine reddened vegetable blues by its immediate contact, and soon after destroyed them.

When the new substance was heated in oxygen gas, or brought in contact with red hot hyperoxymuriate of potassa, it seemed to undergo no change.

M. M. DESORMES and CLEMENT had stated, that, when the substance is combined with the metals, metallic oxides could be obtained from the solutions; I suspected that this depended upon the presence of moisture, or upon oxygen derived from the air, and experiment justified my suspicion.

I heated the substance with iron, mercury, tin, zinc, and lead, out of the contact of air; it united to them without any violence of action, and formed compounds fusible at a moderate heat, and volatile at a higher temperature. All the compounds, except that of zinc, which was white, were coloured of different shades of red brown, red and orange; the compound it formed with tin was of a deep orange, that with iron of a bright red brown, that with lead a bright orange, that with mercury an orange still more approaching to red, and which, when crystallized, was bright crimson.

The compound of iron and the substance, when exposed to an alkaline solution, immediately deposited black oxide of iron;

but when I heated it in a small retort, containing pure ammoniacal gas, no such change occurred, and it combined with the ammonia and formed a compound which volatilized without leaving any oxide.

The compound of the substance with tin was soluble in water, and had the characters of an acid. It combined with the alkalies without depositing oxide.

The crimson compound of the substance with mercury united in the same manner without decomposition to potassa, and by the action of sulphuric acid, sulphate of potassa was formed, and the compound of the substance with mercury disengaged.

When the substance is made to act upon phosphorus, the two bodies combine with great rapidity at common temperatures, producing heat without light; small quantities of a strongly acid gas generally arise from the mixture, and by the application of heat, it is produced in greater quantities. When the substance is in excess, an easily fusible and volatile compound of a red colour is obtained; when the phosphorus is in excess, the greater part of the product is more fixed.

I examined the gaseous acid formed by the action of phosphorus with attention. It gives dense white fumes by combining with the aqueous vapour in the air. It has a smell very similar to that of the solid compound of chlorine and phosphorus, which itself is very analogous to that of muriatic acid. It is rapidly absorbed by water. When made to act upon ammonia, it forms with it a dense white salt, which, when acted upon by sulphuric acid, affords the peculiar substance, and at the same time a smell of hydrogen is perceived. When mercury is heated in the acid gas, the same compound as that

produced by the action of the new substance directly upon mercury is formed, and hydrogen equal to half the volume of the gas is disengaged. When potassium is made to act upon it, there is no inflammation as in muriatic acid gas, but the potassium becomes converted into a body, similar to that produced by its combustion in the vapour of the substance, and a gas equal to half the volume of the acid gas, which burns in the same manner as hydrogen, is disengaged.

When the easily fusible and volatile compound of the substance with phosphorus is heated in water, it rapidly dissolves in it, and forms a strong acid, which, when evaporated, leaves hydrophosphorous acid, and which, before its evaporation, neutralized by potash, and acted on by sulphuric acid, affords the peculiar substance.

When the difficultly fusible substance it forms with phosphorus is acted on by a small quantity of water, and heated in a glass tube, much gas spontaneously inflammable is disengaged, and a white sublimate arises, which, when acted on by cold water, becomes hot, and affords a considerable quantity of a gas having all the properties of hydrophosphoric gas.

The solution of this crystalline substance in water, neutralized by potash, and decomposed by sulphuric acid, afforded the peculiar substance, but when the solution was heated strongly before its neutralization, it left only hydrophosphorous acid, which when heated gave off hydrophosphoric gas, and became phosphoric acid.

It is easy to explain all these phenomena, except the production of the acid gas, which is a compound of the peculiar substance and hydrogen: to account for the appearance of

this body, it is necessary to suppose the existence of hydrogen, or of water in the substance, or of hydrogen in phosphorus.

I used the substance distilled through quick lime, which there is every reason to believe would absorb all the water united to it: in this case the acid gas, which gave hydrogen when decomposed by mercury, was produced in much smaller quantities; but, when the substance was moistened, the gas was afforded in very large quantities. It is probable, that a little hydrogen existing in the phosphorus, and which appears when that substance is acted on by Voltaic electricity, may influence the result; but I am inclined to attribute it principally to the moisture adhering to the substance, and I have never been able to produce more gas from the fusible compound by distilling it with a new quantity of phosphorus.

When the fusible compound of the substance with phosphorus is distilled with a small quantity of water, the gas produced seems to be of the same kind as that obtained by the action of heat during the combination, and both these gases when absorbed by water afford, when acted upon by nitrate of silver, the same product as that formed by the action of a solution of the substance in water on the same salt.

I attempted to form a compound of the substance with hydrogen directly, by heating it in several experiments to redness in a glass tube filled with hydrogen. When the gas was moist, or when the tube contained vapour, a strong acid fluid was formed of a deep yellow colour. When the gas and the substance were dry, there was an expansion of volume, and on breaking the tube, fumes appeared similar to those produced by the action of the gas formed during the union of phosphorus and the substance, and which precipitated in the

same manner a solution of nitrate of silver. This peculiar acid, which consists of the substance united to hydrogen, has a very strong attraction for water, and a very small quantity of water absorbs a large quantity of the gas, and when combined with water, it rises with it in vapour, and in its state of liquid acid, it rapidly dissolves the substance, and becomes tawny.

The new substance, as M. M. DESORMES and CLEMENT have shewn, is rapidly soluble in solution of potash; when it is in excess the solution becomes red brown. On evaporating the mixture and heating it to redness, a substance is formed, exactly similar to that produced by the combination of the substance with potassium.

As potassa is a compound of potassium and oxygen, it is evident that, to form a compound of potassium and the substance from potassa, oxygen must be expelled, and I found by experiment, that this was the case; and in investigating minutely the action of fixed alkaline solutions on the substance, I ascertained the existence of a class of substances, precisely similar to the hyperoxymuriates, consisting of oxygen, the substance, and potassium, and formed in a manner exactly analogous.

If the substance is thrown into a moderately strong solution of potassa as it dissolves, crystals fall down, and by saturating the solution with the substance, considerable quantities are obtained. By pouring off the mother liquor and evaporating it a little, more of the crystals fall down.

All these crystals, if precipitated from a solution not too much saturated, are of the same kind, they are little soluble in water, have a taste analogous to that of the hyperoxymu-

riates of potassa, scintillate when thrown upon burning coals, and form a deflagrating mixture when mixed with charcoal. When fused, they give off abundance of oxygen gas, and become the same substance as that formed by the action of potassium on the new substance.

If the liquor which has ceased to afford the crystals be evaporated to dryness, it yields a considerable quantity of a substance which is not capable of detonating with combustible bodies, and which is the same as that afforded by the combination of the substance with potassium.

It is evident then that the oxygen contained in the potassa is newly combined by the action of the new substance, and two compounds formed, one consisting of potassium and the new substance, the other of potassium, the new substance, and the oxygen contained in the potassa.

By passing the vapour of the substance over dry red hot potassa formed from potassium, oxygen is expelled, and it appears that oxygen cannot remain in the triple compound at a heat above the red heat.

By dissolving the substance in solutions of soda and baryta similar results are obtained, and in each case two compounds are formed. The oxygen is condensed in one, and the other consists simply of the new substance and a metal.

To separate entirely the deflagrating salt from the soluble salt is not easy, there always remains in the mother liquor a little of the deflagrating compound; but by separating the first crystals from solution of potassa not too strong, the deflagrating compound of oxygen, potassium, and the substance is obtained, apparently pure.

As the new substance combines with potassium and the

metals with much less energy than chlorine, it occurred to me, that it would probably be expelled from its combinations by that body; and this I have found to be the case in all the experiments I have made. When the compound of the new substance and potassium is heated in contact with chlorine, potassane (muriate of potassa) is formed, the violet gas appears, but soon combines with chlorine, and they form together the peculiar acid compound I have before described; but towards the end of the process, as the proportion of chlorine diminishes, the violet coloured gas again appears.

When the compound of the substance with silver was treated in the same manner (argentane) muriate of silver was formed, and the substance combined with chlorine at the commencement of the operation, but was disengaged uncombined towards the end.

Similar phenomena occurred when the compounds of the substance with mercury and lead were acted on by chlorine.

The action of acids on the compounds of this substance, are what might be expected from its analogies to chlorine.

When concentrated sulphuric acid is poured upon the compound of the substance and potassium, some of the substance appears; but a part of it rises in combination with hydrogen and water, and condenses by cold, and appears of a deep orange colour from having dissolved some of the substance. The sulphuric acid likewise seems to retain some of the substance, for it continues red after being strongly heated, and the acid is partly decomposed, for sulphurous acid gas is disengaged.

It seems probable, that the acid is decomposed to furnish oxygen to a portion of the potassium which quits in consequence the new substance, and that water is likewise decomposed to

furnish hydrogen to another portion of the substance, and that the hydrogen and the substance, in their acid form, combine with the water of the sulphuric acid, and rise in vapour, sulphate of potassa being at the same time produced.

When sulphuric acid is poured on the salt consisting of the substance, oxygen, and potassium, the substance reappears, and there is a slight effervescence. In this case, part of the oxygen is employed to form potassa, and the remainder is expelled unaltered.

When concentrated nitric acid is made to act on the triple compound, similar phenomena occur, and the substance reappears with effervescence.

When nitric acid is applied to the double compound, there is a smell of nitrous gas, and the substance is instantly reproduced.

With concentrated muriatic acid, the phenomena presented by the two different alkaline compounds, the binary and the triple, are very interesting. When the acid is brought in contact with the triple salt, there is no effervescence, but a substance, which appears to be a compound of chlorine and the new body, is formed and dissolved in the water of the acid, and potassane is precipitated.

When the double compound is used, there is a complete solution with a partial decomposition, and by applying a gentle heat, the excess of muriatic acid is driven off, and the same acid as that procured by the action of the substance on hydrogen remains dissolved in the liquor. When mixtures of the two salts are employed, the substance itself appears.

It appears that in the instance when the triple compound is employed, there is not only sufficient oxygen to attract the hydrogen from the chlorine which is to combine with the

potassium, but likewise enough to decompose a portion of muriatic acid, sufficient to afford chlorine to enter into combination with the whole of the substance.

When the binary compound is used, the result is a simple instance of double affinity; the new substance quits the potassium to unite to the hydrogen of the muriatic acid, and the chlorine and the potassium combine: and that the decomposition is only partial depends probably upon the attraction of the compound of iodine and potassium for water. When mixtures of the compounds are used, the oxygen is employed to attract hydrogen from the substance, to which it seems to adhere with a much weaker attraction than to chlorine.

M. M. DESORMES and CLEMENT have mentioned, that when the new substance is exposed to liquid ammonia, a black powder is formed, which, when dry, fulminates by the slightest contact or friction. I introduced some of the substance into solution of ammonia, and separated the liquor from the black powder, and evaporated it to dryness; it left a white saline substance, the same as that produced by the union of ammonia with the peculiar acid, which consists of the substance combined with hydrogen; and hence it appeared probable that a portion of ammonia had been decomposed to furnish hydrogen.

I made the experiment on the action of strong solution of ammonia on the substance in a pneumatic apparatus; but no azote was given off. Hence I am induced to conclude, that the black powder is a compound of the new substance and azote, similar in its character of a binary compound to the detonating oil discovered by M. DULONG; and this conclusion is strengthened by the results of its detonation in a tube of glass partially exhausted: they are, I find, the peculiar

substance and a gas which is not inflammable, and which does not support flame; and unless the substance is moist, I have never been able to discover any other product; but the minute quantity I have employed prevents me from being confident on this point.

It was an object of considerable interest to ascertain the proportion in which the new substance combines, as compared with that in which the other substances that form acids by their action on inflammable bodies enter into union.

I made several experiments on this subject. 4 grains of hydrate of potassa, I found, were saturated by 6.25 of the new substance, and 2.8 the quantity of potassium in 4 grains of hydrate of potassa is to 6.25 the quantity of the substance, as 75, the number representing potassium, is to 166. Again, 1 grain of hydrate of soda required 2.1 grains of the new substance for its saturation, and 1 grain of hydrate of soda contains .578 of sodium, so that supposing the combination of the new substance with sodium to contain a double proportion of the substance, the number representing the proportion in which it combines will be nearly 160.

Two grains of the compound of the substance with sodium decomposed by sulphuric acid afforded 128 of dry sulphate of soda, and calculating on this experiment the number is 165.5.

I have made some experiments on the quantity of the substance absorbed by tin, mercury, and lead. Mercury absorbs nearly $\frac{1}{4}$ of its weight of the new body to become the crimson substance; from which it appears that it must absorb two proportions.

My experiments have been made upon quantities too small to afford very exact results; but they shew that the new

substance enters into union in a quantity much more than twice as great as that of chlorine, and, considered as an element, it offers a number much higher than those of the simple inflammable bodies, and higher even than those of most of the metals.

The most correct mode of ascertaining the number representing the proportion (supposing it to be definite, as is the case with all other bodies that have been accurately examined) in which it combines, will probably be by ascertaining the specific gravity of its gaseous compound with hydrogen. This gas, as I have stated, affords only half its volume of hydrogen, and it appeared to me to neutralize an equal volume of ammonia, so that supposing it to consist of two proportions of hydrogen, and only one of the substance, that is, to be analogous to muriatic acid gas in its nature, it must be one of the heaviest elastic fluids existing.

Taking the number representing the proportion in which the new substance combines as 165, and supposing that it occupies the same volume in this gas that chlorine occupies in muriatic acid gas, 100 cubical inches of the gas will weigh at mean temperature and pressure 95.27 grains, *i. e.* supposing hydrogen in the same quantity to weigh 2.27 grains.

I am not at present in possession of an apparatus for weighing the gas with accuracy. A particular device will be required for this purpose, as the gas cannot be preserved over mercury. It may be collected during the action of phosphorus on the moistened substance in a vessel exhausted of air; or it may be made by heating the compound of the substance and potassium in muriatic acid gas in a glass vessel: in this case, there is, I find, a double decomposition, the chlorine quits the

hydrogen to unite to the potassium, and the substance quits the potassium to unite to the hydrogen.

The new substance, I find, is not decomposed when Voltaic sparks are taken in it in its gaseous state from ignited points of charcoal: at first there are white fumes, probably from the action of moisture or hydrogen in charcoal, on the substance; but these fumes soon cease, and when the tube in which the experiment is made is cooled, the substance appears unaltered.

From all the facts that have been stated, there is every reason to consider this new substance as *an undecompounded body*. In its specific gravity, lustre, the high number in which it enters into combination and colour, it resembles the metals; but in all its chemical agencies it is more analogous to oxygen and chlorine; it is a non-conductor of electricity, and possesses, like these bodies, the negative electrical energy with respect to metals, inflammable and alkaline substances, and hence when combined with these substances in aqueous solution and electrized in the Voltaic circuit, it separates at the positive surface; but it has a positive energy with respect to chlorine, for when united to chlorine in the compound acid I have described, (page 78), it separates from the chlorine at the negative surface. This likewise corresponds with their relative attractive energy. Chlorine expels the new substance from all its combinations on which I have made any experiments.

The new substance seems to possess a stronger attraction for most of the metals than oxygen; but it is expelled from phosphorus and sulphur by oxygen: I found by passing oxygen and the compound of it with phosphorus through a glass tube heated red, phosphorous acid was formed, and the violet gas appeared.

That it produces so little heat and so seldom light in entering into combination, may be accounted for from its solid form and its great weight as an element. Potassium, however, as I have mentioned, burns in the violet coloured gas, and when this gas is thrown upon the flame of hydrogen, it seems to support its combustion.

The saturating or neutralising powers of the new substance appear to be greater than those of oxygen, and less than those of chlorine.

It agrees with chlorine and fluorine in forming acids with hydrogen, and it agrees with oxygen in forming an acid with chlorine.

In my first experiments I conceived that it formed substances analogous to alkalies in combining with the alkaline metals, for the compound produced by its action upon solution of potassa, even when the substance was in great excess, reddened turmeric paper, and rendered green paper tinged with the juice of violets; but I have since found that this is owing to a small quantity of subcarbonate of potassa which existed in the hydrate of potassa; and when the compound is treated with the acid the substance forms with hydrogen, and heated to redness, it loses this property; and when thus formed, its taste more resembles that of a neutral salt than of an alkali. I cannot yet say with certainty whether its compound with potassium has powers like the oxides, of neutralising those acids which it does not decompose, as in all the experiments I have made on this point I used the compound which reddens turmeric; this neutralised the phosphorous, sulphurous, and boracic acids, but the effect may possibly depend upon the undecomposed carbonate.

The name *ione* has been proposed in France for this new substance from its colour in the gaseous state, from ἴον, viola; and its combination with hydrogen has been named *hydroionic acid*. The name *ione*, in English, would lead to confusion, for its compounds would be called *ionic* and *ionian*. By terming it *iodine*, from ἰώδης, violaceous, this confusion will be avoided, and the name will be more analogous to chlorine and fluorine.

The acid it forms with hydrogen may, however, be with propriety named in our language *hydroionic acid*. I venture to propose for the acid it forms with chlorine, the name of *chlorionic acid*, and for that it forms with tin *stannionic acid*. With respect to the other compounds, they may be called as a class *iodes*, with the name of the base as *iode of mercury*, and with *proto*, *deuto*, &c. to signify the proportions; or if a termination to the base should be preferred, as I have proposed for the combinations of chlorine, the terminations may be in *m*, with the vowels in their usual order to signify proportions. Thus, phosphoroma would signify the combination of one proportion of iodine with phosphorus, and phosphoromè would signify two proportions of iodine to one of phosphorus.

If this last plan, which involves no theoretical views, should be adopted, it may be extended with different consonants to the combinations of *fluorine*, and the vowel may be made to signify the proportion, and the consonant the nature of the compound. The vowel of termination, to the Latin name of the base, I have already proposed, on another occasion, for the compounds of oxygen. Thus argenta may be made to signify the protoxide of silver, and ferrè the deutoxide of iron. *n* is the consonant which I have suggested to

represent the combination of chlorine, as *argentana* the *protochloride* of silver; and *l* in this system may represent fluorine. Thus *calcala* would be fluor spar, or one proportion of fluorine, and one of calcium, and the different combinations of calcium with the supporters of combustion with oxygen, fluorine, chlorine, and iodine in one proportion, would be thus expressed *calca*, *calcala*, *calcana*, and *calcama*.

I throw out these hints for discussion, rather than with any wish for their adoption, and for the purpose of directing the attention of chemists towards the subject of nomenclature, which ought to be settled on some fixed principles; and in naming a new class of compounds, great caution should be used to prevent the necessity of alteration.

In my last paper, presented to the Society two months ago, I ventured to suggest that it was probable, that new species of matter, which act with respect to inflammable bodies, like oxygen, chlorine, and fluorine, would be discovered. I had not hoped, at that time, to be able so soon to describe the properties of a body of this kind, which forms an acid with hydrogen, like chlorine and fluorine, and which in some of its combinations resembles oxygen.

This new fact will, I hope, do something towards settling the opinion of chemists respecting the nature of acidity, which seems to depend upon peculiar combinations of matter, and not on any peculiar elementary principle.

It is probable that *iodine* will be found in many combinations in nature. We may expect that it will be discovered in various marine vegetables, and in sea water, and probably the loss of weight indicated in the analysis of certain fossil substances may depend upon its expulsion.

Its compounds with the metals will probably form a new class of pigments, and it is not impossible, that the triple salts it forms containing oxygen, may be made substitutes for nitre in the manufacture of gunpowder.

Paris, Dec. 10, 1813.

VII. *An Account of a Family having Hands and Feet with supernumerary Fingers and Toes.* *By* Anthony Carlisle, *Esq. F. R. S. In a Letter addressed to the Right Hon. Sir* Joseph Banks, *Bart. K. B. P. R. S.*

Read December 23, 1813.

Dear Sir,

The following account of a family having hands and feet with supernumerary fingers and toes, and the hereditary transmission of the same peculiarity to the fourth generation, appears to be worth preserving, since it displays the influence of each of the propagating sexes; the male and the female branches of the original stem having alike reproduced this redundancy of parts. I have carefully inspected two persons of this family at the time of their being in London, namely, Abiah Colburn, and his son Zerah Colburn, and have taken the particulars of the rest from Abiah Colburn himself, whose narrative was several times repeated to me, without any deviation.

Zerah Colburn, a native of the township of Cabot in the province of Vermont, in North America, has been lately brought to London, and publicly exhibited for his extraordinary powers in arithmetical computations from memory. This boy has a supernumerary little finger growing from the outside of the metacarpus of each hand, and a supernumerary little toe upon the outside of the metatarsus of each foot.

These extra fingers and extra toes are all completely formed, having each of them three perfect phalanges with the ordinary joints, and well shaped nails.

ABIAH COLBURN, the father of ZERAH, has five fingers and a thumb upon each hand, and six toes on each foot; he has also five metacarpal bones in each hand, and six metatarsal bones in each foot. The extra limbs have distinct flexor and extensor tendons.

The wife of ABIAH COLBURN has no peculiarity in her limbs. During the existing marriage, she has borne eight children, six sons, and two daughters. Four of those sons inherit the peculiarity of their father more or less complete, while the two daughters are free from the family mark, as well as two of the sons, namely, the fourth in succession who was a twin, and the eighth.

The eldest son of these parents, named GREEN COLBURN, has only five toes on one of his feet, but the other foot, and both his hands possess the extra limb.

The second child, BETSY COLBURN, is naturally formed.

The third, ZEBINA COLBURN, has five fingers and a thumb upon each hand, and six toes upon each foot.

The fourth and fifth were twin brothers, and named DAVID and JONATHAN; DAVID, who is dead, had nothing of the father's mark, but,

JONATHAN has the peculiarity complete.

The sixth, ZERAH COLBURN, the extraordinary calculating boy, is marked like his father, as before described.

The seventh, MARY COLBURN, is naturally formed.

The eighth and last child, ENAS COLBURN, is also exempt from the father's peculiarity.

Besides the persons I have mentioned, this hereditary redundance of limbs has been attached to the little fingers, and to the little toes of several of the ancestors of the family. The mother of ABIAH COLBURN brought the peculiarity into his family. Her maiden name was ABIGAIL GREEN; she however had not the extra finger on one of her hands; the other hand, and her feet were similarly marked with those of her son ABIAH.

DAVID COLBURN, the father of ABIAH, had no peculiarity. By his marriage with ABIGAIL GREEN, he had three sons and one daughter. Two of these sons and the daughter were fully marked in all the limbs; the other son had one hand and one foot naturally formed.

ABIGAIL GREEN inherited these supernumerary limbs from her mother, whose maiden name was —— KENDALL, and she had five fingers and a thumb upon each hand, and six toes on each foot.

The marriage of —— KENDALL with Mr. —— GREEN produced eleven children, whom ABIAH COLBURN'S mother, who was one of the eleven, reports to have been all completely marked: but the present family are unacquainted with the history of the other ten branches, and they do not possess any knowledge of their ancestors beyond —— KENDALL, the great grandmother of ZERAH COLBURN.

Numerous examples of the hereditary propagation of peculiarities have been recorded: all family resemblances, indeed, however trifling they may appear to a common observer, are interesting to the physiologist, and equally curious; though not so rare as those described in the preceding history. In every department of animal nature, accumulation of facts must

always be desirable, that more reasonable inductions may be established concerning the laws which direct this interesting part of creation: and it might be attended with the most important consequences, if discovery could be made of the relative influence of the male and female sex in the propagation of peculiarities, and the course and extent of hereditary character could be ascertained, both as it affects the human race in their moral and physical capacities, and as it governs the creatures which are subdued for civilized uses. Nor is it altogether vain to expect, that more profound views, and more applicable facts await the researches of men, who have as yet only begun to explore this branch of natural history, by subjecting it to physical rules.

Though the causes which govern the production of organic monstrosities, or which direct the hereditary continuance of them, may for ever remain unknown, it still seems desirable to ascertain the variety of those deviations, and to mark the course they take, where they branch out anew, and where they terminate. There is doubtless a general system in even the errors of nature, as is abundantly evinced by the regular series of monstrosity exhibited both in animals and vegetables.

It has happened in my professional capacity, that I have had to extirpate a supernumerary thumb from each of the hands of two girls, who were both ideots, though the families to whom they belonged were unknown to each other. I have seen many instances of supernumerary thumbs and supernumerary fingers in persons to whom the singularity was not hereditary, and I have read of many others; but whether of my own experience, or of authentic record, the redundancy has

been on the outer side of the little finger, and outer side of the thumb, never on the back or inside of the hand, or on the sides of the intermediate fingers: and in similar cases as to the toes, the rule has been invariably the same. In the Sacred Writings an example of this kind is given, II Samuel, ch. 21. v. 20. "And there was yet a battle in Gath, where was a man "of *great* stature, that had on every hand six fingers, and on "every foot six toes, four and twenty in number; and he also "was born to the giant." The same account is repeated in I Chronicles, ch. 20. v. 6.

In the Elementa Physiologiæ of Baron HALLER, numerous examples of this deformity are cited from various authors, with some instances of their hereditary descent, and others of a cutaneous junction between the extra limbs and the next adjoining.*

That local resemblances, such as those of external parts, the hands, the feet, the nose, the ears, and the eye-brows, are hereditary, is well known; and it is almost equally evident, that some parts of the internal structure are in like manner transmitted by propagation: we frequently see a family form of the legs and joints, which gives a peculiar gait, and a family character of the shoulders, both of which are derived from an hereditary similarity in the skeletons. Family voices are also very common, and are ascribable to a similar cause. Apparently many of our English surnames have been taken from the hereditary peculiarities of families, and the same practice existed among the Romans. PLINY, in his eleventh Book, chap. 48, relates an instance of a Roman poet, named VOLCATIUS, who had six fingers on each hand, and received the

* Vide Vol. viii. p. 98.

surname of SEDIGITUS in consequence. He also states, that two daughters of a noble Roman, named M. CURIATIUS, had each six fingers, and that they took the surname of SEDIGITÆ. Persons who had the surname of FLACCUS were so called from their pendulous ears, and numerous other instances are recorded by classic writers of surnames being derived from family marks.

Anatomical researches have not been so generally extended as to determine the prevalence of internal peculiarities, and perhaps they do not reach to the sanguineous system. I have known two instances, in two different families, of the high division of the brachial arteries having the ulnar branch placed above the fascia of the biceps muscle at the inner bend of the elbows, and yet the father, the mother, the brothers and sisters of those two persons were not so formed. Those marks called nævi materni, which are derangements of the sanguineous vessels, are not hereditary, whilst less remarkable changes in the ordinary skin are often so. I have lately seen a man, and who is now living, who has a small pendulous fold attached to the skin of his upper eyelid, and the same peculiarity has been transmitted to his four children. It would have been interesting to know, whether any similarity of structure existed in the families of the two rare examples of a total transposition of the abdominal and thoracic viscera. (Phil. Trans. for 1674, No. 107. pag. 146, by Dr. SAMPSON, and Vol. 78, page 350.)

In particular breeds of animals, the characteristic signs are generally continued, whether they belong to the horns of kine, the fleeces of sheep, the proportions of horses, the extensive varieties of dogs, or the ears of swine. In China the varieties

of gold and silver fishes are carefully propagated, and with us, what are vulgarly called "fancy pigeons" are bred into most whimsical deviations from their parent stock.

As wild animals and plants are not liable to the same variations, and as all the variations seem to increase with the degree of artificial restraint imposed, and as certain animals become adapted by extraordinary changes to extraordinary conditions, it may still be expected that some leading fact will eventually furnish a clue, by which organic varieties may be better explained. A few generations of wild rabbits, or of pheasants under the influences of confinement, break their natural colours, and leave the fur and feathers of their future progeny uncertainly variegated. The very remarkable changes of the colour of the fur of the hare, and the feathers of the partridge in high northern latitudes, during the prevalence of the snow, and the adaptation of that change of colour to their better security, are coincidences out of the course of chance, and not easily explained by our present state of physical knowledge.

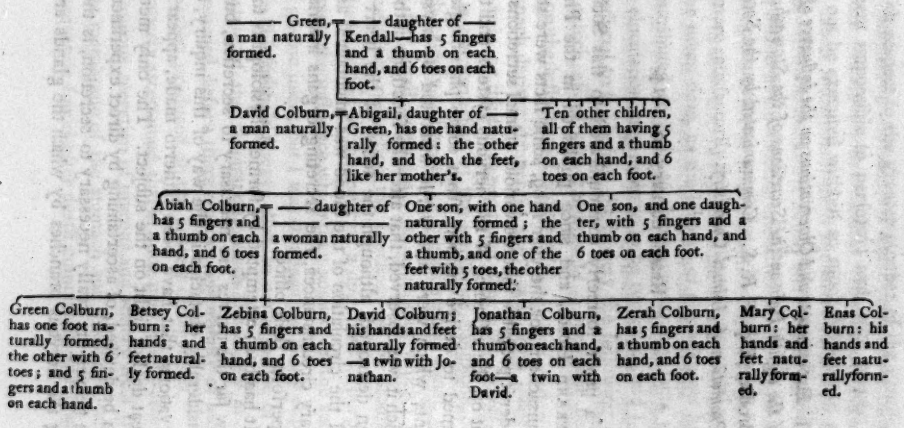
I have the honour to be, Dear Sir,

your much obliged and obedient servant,

ANTHONY CARLISLE.

To the Right Hon. Sir Joseph Banks, Bart.
K. B. P. R. S. &c.

A Table shewing the hereditary Descent of this Peculiarity, according to the Method of Genealogists.


—— Green, a man naturally formed.
—— daughter of —— Kendall—has 5 fingers and a thumb on each hand, and 6 toes on each foot.
David Colburn, a man naturally formed.
Abigail, daughter of —— Green, has one hand naturally formed: the other hand, and both the feet, like her mother's.
Ten other children, all of them having 5 fingers and a thumb on each hand, and 6 toes on each foot.
Abiah Colburn, has 5 fingers and a thumb on each hand, and 6 toes on each foot.
—— daughter of —— —— a woman naturally formed.
One son, with one hand naturally formed; the other with 5 fingers and a thumb, and one of the feet with 5 toes, the other naturally formed.
One son, and one daughter, with 5 fingers and a thumb on each hand, and 6 toes on each foot.
Green Colburn, has one foot naturally formed, the other with 6 toes; and 5 fingers and a thumb on each hand.
Betsey Colburn: her hands and feet naturally formed.
Zebina Colburn, has 5 fingers and a thumb on each hand, and 6 toes on each foot.
David Colburn; his hands and feet naturally formed—a twin with Jonathan.
Jonathan Colburn, has 5 fingers and a thumb on each hand, and 6 toes on each foot—a twin with David.
Zerah Colburn, has 5 fingers and a thumb on each hand, and 6 toes on each foot.
Mary Colburn: her hands and feet naturally formed.
Enas Colburn: his hands and feet naturally formed.

VIII. *Experiments and Observations on the influence of the Nerves of the eighth Pair on the Secretions of the Stomach. By* B. C. Brodie, *Esq. F. R. S. Communicated by the Society for the Promotion of Animal Chemistry.*

Read February 10, 1814.

In a paper formerly communicated to this Society by Sir Everard Home, and since published in the Philosophical Transactions for the year 1809, some facts were stated which render it probable that the various animal secretions are dependent on the influence of the nervous system, and this opinion seemed to derive support from some physiological experiments which were afterwards instituted by myself, and in which it was observed, that after the functions of the brain had been destroyed, although the action of the heart continued, and the circulation of the blood was maintained as under ordinary circumstances, the secreting organs invariably ceased to perform their office.

It has been attempted by former physiologists to determine how far the nerves are necessary to secretion, but there are considerable obstacles in the way of this inquiry, and no observations, that have been hitherto made, appear to throw a great deal of light on the subject. The only method, which can be devised, of ascertaining by direct experiment, whether the nerves are really necessary to secretion, is that of dividing the nervous branches by which the glands are supplied.

But this, with respect to the greater number of the glands, is an experiment impossible to perform; and, with respect to others, can not be executed without so much disturbance and injury to the other parts, as must render it extremely difficult to arrive at any positive results. Perhaps in future investigations, some circumstances may arise, which will enable us to determine more satisfactorily this important physiological question. In the mean time, as the labours of physiologists have hitherto contributed so little to this purpose, any facts that tend to its elucidation may deserve to be recorded, and I am therefore induced to lay before the Society the following experiments, which afford one example of a secretion being dependent on the influence of the nerves.

The stomach derives its nerves principally from those of the eighth pair, or the *par vagum*; and the same nerves, as they assist in the formation of the semilunar ganglions, contribute to the supply of the rest of the alimentary canal, particularly of the small intestines. In an inquiry which I had formerly instituted, respecting the functions of the stomach, I divided these nerves in the neck of a dog, for the purpose of ascertaining the influence which they possess on the secretion of the gastric juice; but I was disappointed in my expectation, since the animals always died, in consequence of the disturbed state of the respiration, which the injury of the nerves occasioned, before there was an opportunity of ascertaining the effect produced on the process of digestion.

I had formerly ascertained, that in a dog poisoned by arsenic, there is a copious secretion of mucous and watery fluid from the mucous membrane of the stomach and intestines, which are in consequence found after death completely and preter-

naturally distended; and it occurred to me, that although I could not ascertain the effect of the division of the nerves of the eighth pair on the natural secretions of the stomach, it might be possible to ascertain the effect on a secretion thus artificially produced. With this view I instituted the following experiments.

Exp. 1. The nerves of the eighth pair with the accompanying sympathetic nerves, were divided in the neck of a dog, and immediately afterwards ten grains of arsenic were inserted into a wound of the thigh. The breathing became laborious, as is usual where these nerves are divided, and afterwards the same symptoms took place, as commonly arise from the poison of arsenic, with this difference, that there was no discharge of fluid either from the stomach or intestines. He died at the end of three hours and a half. On dissection, the stomach and intestines were found to contain only food and fæces, there being none of the mucous and watery secretion usually met with in an animal which has been killed in the same manner. The mucous membrane of the stomach and intestines was highly inflamed.

Exp. 2. The experiment was repeated on another dog. He died at the end of nine hours, and on dissection the stomach and intestines were not found to contain any mucous or watery fluid. Their mucous membrane was inflamed.

Exp. 3. A dog, immediately after the division of the same nerves in the neck, was made to swallow two ounces of saturated solution of white oxide of arsenic in water. He died at the expiration of three hours: on dissection, the stomach and intestines were found slightly inflamed, and they contained no mucous or watery fluid.

In these experiments, the animals died from the application of the arsenic, and the poison produced the usual symptoms, with the exception of the copious mucous secretion, which takes place in other instances from the stomach and intestines. The obvious conclusion was, that this secretion was prevented in consequence of the nervous influence having been interrupted by the division of the nerves of the eighth pair; but as this injury always induces a disturbed and laborious respiration, it was desirable to ascertain how far this circumstance might have operated towards the production of this effect, and I therefore repeated the experiment, but with this difference, that the nerves were divided in such a way as not to interfere with the functions of the lungs.

Exp. 4. Having made an incision into the abdomen of a dog, immediately below the short ribs, I divided, by means of a bistoury, the stomachic ropes formed by the nerves of the eighth pair, where they are situated on the œsophagus, immediately above the cardiac orifice of the stomach. The wound was closed by sutures. The respiration was not in the least disturbed, but was performed as frequently, and with as much ease, as under ordinary circumstances. The animal was afterwards inoculated in the thigh with the white oxide of arsenic, and he died in a few hours after the application of the poison, with the ordinary symptoms, except that there were no fluid evacuations from the stomach or intestines.

On dissection, the mucous membrane of the stomach and intestines was found inflamed. There was no watery or mucous fluid in the stomach or small intestines. There was a small quantity of mucus in the colon.

The result of this being the same as that of the former

experiments, we may conclude that the suppression of the secretions in all of them was to be attributed solely to the division of the nerves: and all the facts which have been stated sufficiently demonstrate, that the secretions of the stomach and intestines are very much under the controul of the nervous system. We cannot indeed venture to deduce from them any positive conclusions respecting the necessity of the nervous influence to the secretions in general, but as forming one link in the chain of an interesting, but difficult physiological investigation, the circumstances which have been mention may perhaps be considered as possessing some value, and as worthy of being recorded.

It is proper to observe, that I have attempted to pursue the investigation, so as to ascertain the effect produced on the process of digestion by the division of the stomachic ropes on the termination of the œsophagus, but various circumstances, which it would be unnecessary to enumerate, have prevented my proceeding in the inquiry, and seem almost to render it impossible, to make any observations on this subject.

IX. *On a fossil human Skeleton from Guadaloupe.* By Charles Konig, *Esq. F. R. S. In a Letter addressed to the Right Hon. Sir* Joseph Banks, *Bart. K. B. P. R. S.*

Read February 10, 1814.

My Dear Sir,

British Museum, Dec. 20, 1813.

The human skeleton imbedded in limestone, lately brought from Guadaloupe by the Hon. Sir Alexander Cochrane, and presented by the Admiralty to the British Museum, having excited the curiosity of the public, I do myself the honour of submitting to you a short account of these fossil remains, which, (though fully aware of the weight of those arguments you lately urged in conversation, as unfavourable to the probability of their high antiquity) I am still led to consider as not altogether uninteresting to the geologist. The neglect of the more recent formations of rocks was one principal cause that the nature and origin of the older had been so strangely misunderstood: it is to the more attentive observation that has of late years been paid to the phenomena which they present, that we are indebted for the knowledge of the important fact, that secondary fossils are the productions and depositions of various periods; by which happy discovery the study of these remains, formerly a puerile and frivolous pursuit, has become a most important branch of natural science. The osseous relics of a later period are more particularly calculated to furnish data for the future construction of a philosophical

theory, at least of the crust of the earth. It appears therefore strange, that these latter, although they speak a far more intelligible language than the rest, should, till within a very short period, have been almost entirely disregarded by the speculative geologist. Bones, the most distinctly organized, were so very imperfectly understood, even in the beginning of the last century, that Dr. CARL, in his *Ossium fossilium docimasia*, published at Frankfurt in 1704, found it necessary to employ all his powers of reasoning, aided by chemical analysis, to prove that they were neither *lusus naturæ*, nor the results of a *vis plastica* of the earth. At a still later period, the principal value put upon the discovery of these fossil bones, was derived from the supposed proof they afforded of the former existence of a gigantic race of men, or from the evidence they were supposed to give in behalf of an universal deluge: and it is curious to observe, to what a degree men eminent for science, but very deficient in critical acumen, have been led astray in their speculations, under the influence of preconceived notions.

CAMPER and BLUMENBACH were among the first who thought of connecting the subject of osseous fossil remains with systematic geology. The former of these naturalists had, at first, controverted the doctrine of the extinction of genera and species as incompatible with divine providence; but, afterwards adopting it himself, he alludes to a prior creation to that of man, in the following passage, which, I suppose, has never been quoted: "Hodie, quam plurima extinctorum (animalium) "specimina in Museo meo reperiunda, et meditationes magis "seriæ, persuaserunt mihi, sapientiæ Divinæ non repugnare "legem, qua res illas vel animalia illa desinere jubeat, simulac

" scopo primario nobis incognito satisfecerunt penitus. Con-
" victus etiam quam maxime sum, orbem nostrum variis illis
" ac horrendis catastrophis fuisse expositum aliquot seculis
" antequam homo fuit creatus: nunquam enim hucusque videre
" mihi contigit verum os humanum petrifactum, aut fossile,
" etiamsi Mammonteorum, Elephantorum, Rhinocerotum,
" Bubalorum, aliorumque perplurima viderim ossa." Nova Act. Petrop. Tom. II. p. 251.

But on the history of the strata produced by the more recent catastrophes of the globe (and it is the formation of these alone which can be scrutinized with any reasonable prospect of success) most light has been thrown by the indefatigable exertions of M. Cuvier. Superlatively skilled in comparative anatomy, this gentleman has succeeded in determining the fossil bones of no less than seventy-eight species, of which forty-nine are entirely unknown among the existing race of animals; about twelve are identified with known species, and the remainder strongly resemble existing species, although their identity has not been completely ascertained. From the multiplied observations which this naturalist has communicated in his numerous memoirs, we may gather that the viviparous quadrupeds appear at a much later period in the fossil state than the oviparous; the latter being probably coeval with the fishes, whilst the former are found only in the newest formations, in which, according to Brongniart and Cuvier's interesting discovery, marine beds are observed to alternate with those of fresh water, and which (in the neighbourhood of Paris) overlay the coarse shell limestone which constitutes the last strata formed, as it would appear, by a long and quiet stay of the sea on our continent.

All the circumstances under which the known depositions of bones occur, both in alluvial beds and in the caverns and fissures of Fletz limestone, tend to prove, that the animals to which they belonged met their fate in the very places where they now lie buried. Hence it may be considered as an axiom, that man, and other animals whose bones are not found intermixed with them, did not co-exist in time and place. The same mode of reasoning would further justify us in the conclusion that, if those catastrophes, which overwhelmed a great proportion of the brute creation, were general, as geognostic observations in various parts of the world render probable, the creation of man must have been posterior to that of those genera and species of mammalia, which perished by a general cataclysm, and whose bones are so thickly disseminated in the more recent formations of rocks.

The many instances of Anthropolithi described by authors, from SCHEUCHZER's famous "*Homo diluvii testis et* Θεοπιστος," to SPALLANZANI's mountain of human bones in the island of Cerigo, have all proved not to be what they were taken for by the ignorant in osteology, and cannot, therefore, be adduced in objection to the above reasoning. Still less are the incrustations of human bones, (from the once celebrated skeleton preserved in the Villa Lodovisi at Rome, to the skull found in the Tiber, and preserved in the British Museum,) calculated to subvert it; nor is the hypothesis likely to be greatly invalidated by the subject of this letter, with whose history, however, several circumstances are connected which appear to preclude the probability of a very recent deposition, or which, at least, require further elucidation, before its relative age can be pronounced upon with any degree of confidence.

The human skeletons from Guadaloupe are called *Galibi* by the natives of that island; a name said to have been that of an ancient tribe of Caribs of Guiana; but which, according to your plausible conjecture, originated in the substitution of the letter *l* instead of *r* in the word *Caribee*. I find no mention made of them by any author, except General Ernouf, in a letter to M. Faujas St. Fond, inserted in Vol. V. (1805) of the *Annales du Muséum*; and by M. Lavaisse, in his *Voyage à la Trinidad*, &c. published in 1813. The former of these gentlemen writes, that on that part of the windward side of the Grande-Terre, called *La Moule*, skeletons are found enveloped in what he terms "*Masses de Madrépores pétrifiés*," which being very hard, and situated within the line of high water, could not be worked without great difficulty, but that he expected to succeed in causing some of these masses to be detached, the measurements of which he states to be about eight feet by two and a half.

The block brought home by Sir Alex. Cochrane exactly answered this account, with regard to the measurements; its thickness was about a foot and a half. It weighed nearly two tons; its shape was irregular, approaching to a flattened oval, with here and there some concavities, the largest of which, as it afterwards appeared, occupying the place where the thigh bone had been situated, the lower part of which was therefore wanting. Except the few holes evidently made to assist in raising the block, the masons here declared that there was no mark of a tool upon any part of it; and indeed the whole had very much the appearance of a huge nodule disengaged from a surrounding mass.

The situation of the skeleton in the block was so superficial,

that its presence in the rock on the coast had probably been indicated by the projection of some of the more elevated parts of the left fore-arm.

The operation of laying the bones open to view, and of reducing the superfluous length of the block at its extremities, being performed with all the care which its excessive hardness and the relative softness of the bones required, the skeleton exhibited itself in the manner represented in the annexed drawing (Pl. III.), with which my friend Mr. ALEXANDER, has been so good as to illustrate this description.

The skull is wanting; a circumstance which is the more to be regretted, as this characteristic part might possibly have thrown some light on the subject under consideration, or would, at least, have settled the question, whether the skeleton is that of a Carib, who used to give the frontal bone of the head a particular shape by compression, which had the effect of depressing the upper and protruding the lower edge of the orbits, so as to make the direction of their opening nearly upwards, or horizontal, instead of vertical.*

The vertebræ of the neck were lost with the head. The bones of the thorax bear all the marks of considerable concussion, and are completely dislocated. The seven true ribs of the left side, though their heads are not in connexion with the vertebræ, are complete; but only three of the false ribs are observable. On the right side only fragments of these bones are seen; but the upper part of the seven true ribs of this side are found on the left, and might at first sight be taken for the termination of the left ribs; as may be seen in the drawing. The right ribs must, therefore, have been violently

* See the excellent figures in BLUMENBACH's *Decades*.

broken and carried over to the left side, where, if this mode of viewing the subject be correct, the sternum must likewise lie concealed below the termination of the ribs. The small bone dependent above the upper ribs of the left side, appears to be the right clavicle. The right os humeri is lost; of the left nothing remains except the condyles in connexion with the fore-arm, which is in the state of pronation: the radius of this side exists nearly in its full length, while of the ulna the lower part only remains, which is considerably pushed upwards. Of the two bones of the right fore-arm, the inferior terminations are seen. Both the rows of the bones of the wrists are lost, but the whole metacarpus of the left hand is displayed, together with part of the bones of the fingers: the first joint of the fore-finger rests on the upper ridge of the os pubis, the two others, detached from their metacarpal bones, are propelled downwards, and situated at the inner side of the femur and below the foramen magnum ischii of this side. Vestiges of three of the fingers of the right hand are likewise visible, considerably below the lower portion of the fore-arm, and close to the upper extremity of the femur. The vertebræ may be traced along the whole length of the column, but are in no part of it well defined. Of the os sacrum, the superior portion only is distinct: it is disunited from the last vertebra and the ilium, and driven upwards. The left os ilium is nearly complete, but shattered, and one of the fragments depressed below the level of the rest: the ossa pubis, though well defined, are gradually lost in the mass of the stone. On the right side, the os innominatum is completely shattered, and the fragments are sunk; but towards the acetabulum, part of its internal cellular structure is discernible.

The thigh bones and the bones of the leg of the right side are in good preservation, but being considerably turned outwards, the fibula lies buried in the stone, and is not seen. The lower part of the femur of this side is indicated only by a bony outline, and appears to have been distended by the compact limestone that fills the cavities both of the bones of the leg and thigh, and to the expansion of which these bones probably owe their present shattered condition. The lower end of the left thigh bone appears to have been broken and lost in the operation of detaching the block; the two bones of the leg, however, on this side, are nearly complete: the tibia was split almost the whole of its length a little below the external edge, and the fissure being filled up with limestone, now presents itself as a dark coloured straight line. The portion of the stone which contained part of the bones of the tarsus and metatarsus was unfortunately broken; but the separate fragments are preserved.

The whole of the bones, when first laid bare, had a mouldering appearance, and the hard surrounding stone could not be detached without frequently injuring their surface; but after an exposure for some days to the air, they acquired a considerable degree of hardness. Sir H. DAVY, who subjected a small portion of them to chemical analysis, found that they contained part of their animal matter, and all their phosphate of lime.

The calcareous rock in which these bones are imbedded, is an aggregate, composed principally of zoophytic particles, and the detritus of compact limestone: it readily dissolves in diluted nitric acid, without leaving any evident residue. Its general colour is greyish yellow, passing into yellowish grey.

When more closely examined, it is found to consist of yellowish grains, intermixed with others of a more or less deep flesh red colour. These grains, though minute, are in some parts of the mass perfectly defined, and in close contact with each other, although no cement is perceptible; in other parts they are, as it were, confluent, forming a more or less porous mass; in others again they form a compact mass, in which the former distinct concretions, especially the red ones, are only indicated by a difference of colour.

The red grains that enter the composition of the rock, appeared at first view to be particles of red coral (isis nobilis), which has hitherto been found only in the Mediterranean; but on closer examination, their structure proved them to be the detritus of a millepora, and indeed one fragment of a larger size than the rest renders it probable that it is *millepora miniacea* of PALLAS. The last mentioned fragment was coalesced with a piece of very compact white madrepore, and intimately connected with the surrounding mass. Shells also are found in this rock: the one most distinct is a helix, approaching in form and size to *helix acuta* of MARTINI, (Conchiol. Vol. IX. Pl. XXX. fig. 224.), but differing in the form of the whorls which are less convex and distinct, and have three bands on the body volution, instead of one. Another shell, of which a few only were discovered in the mass, appears to be *turbo pica* of LINNÆUS in a worn state: the brown spots are still distinctly seen on its surface. Dr. LEACH intends to dedicate a plate of one of the next numbers of his "*Zoological Miscellany*" to the illustration of these and some other shells related to them.

Besides these bodies, I found, near the surface of the block,

part of a bone of a concentric lamellated structure, apparently the fragment of a tusk, but of what animal I am unable to determine; also a large fragment of a basaltic stone; and here and there small nests and dots of a black powdery substance, which, from its decomposing nitre with great energy, appears to be pure charcoal.

The hardness of this limestone, as calculated by the degree of impression made upon it by the mason's saw and chisel, surpasses that of statuary marble.

From this description of the rock, it will be sufficiently clear, that it is by no means of a stalactical nature, and cannot therefore be compared either with travertino, or any other chemical calcareous deposition of this kind. Its origin seems unquestionably to be similar to that of common sandstone, only that the grains of which it is composed have in some parts become confluent, and formed a nearly compact limestone. Whether this last circumstance be not in part owing to the admixture of the gelatinous matter of the bones and muscular parts of the body of which it is the remains, I am not prepared to decide; certain it is, that the mass approaches more to the compact state in the immediate vicinity of the bones, than at a distance from them. Dr. THOMSON informs me that he has found traces of phosphate of lime in this stone.

Respecting the age of these fossil remains, if not much positive information can be derived from the preceding details, they will prove at least, that the enveloping rock is not of a stalactic nature, and that the bones, after they were deposited, underwent a degree of violence which dislocated and fractured them, without removing the fragments to a distance from each

other. It may therefore be safely concluded, that the surrounding mass must have been in a soft or semi-fluid state, which, whilst it opposed no effectual resistance to a shock from without, readily filled up the chasms produced by it.

From the composition of the stone, a late period may, perhaps, be assigned to its formation; yet there is nothing in the above description that necessarily implies a very recent origin. For although there are many instances of gravel and sand being quickly formed into hard masses; and even art has availed itself of this circumstance to produce from the granitic detritus a complete regenerated granite (in which cementation of loose siliceous grains oxyd of iron is well known to be a powerful agent), yet we know of no limestone being formed as it were under the eyes of men; for stalactically concreted limestone, as I have already observed, should not be confounded with this.

Saussure (as you had the goodness to point out to me) mentions, indeed, sands on the shore near Messina, which, by means of a calcareous juice from the sea, as that writer expresses himself, acquire a degree of hardness, which renders them fit to serve for mill-stones; but it would appear, from the context, that the sand thus agglutinated is siliceous.

The circumstance of these bones not being actually petrified, and even retaining part of their gluten, though considered by some as a proof of their recent deposition, is by no means conclusive; for there does not seem to be any reason why lapidification of organic bodies should ever take place, under circumstances unfavourable to that remarkable process: accordingly, the bones in the Fletz limestone caverns, and in the breccia of Gibraltar, Dalmatia, Cette, &c. appear not to have

made the least progress towards a petrified state. Most of the ivory employed in Russia is derived from the tusks of the fossil extinct species of elephant; and even the osseous remains of the plaister quarries of Paris, according to VAUQUELIN's analysis, have yielded animal matter. Shells also may, for a considerable lapse of time, escape the influence of petrifying causes; a remarkable instance of which we have in the calcareous rock of Maestricht, which, together with the remains of extinct species even of amphibious animals, encloses shells but very little altered from their original state.

The attention of geologists being now directed towards this object, it may be expected that a scientific examination of the circumstances under which this limestone occurs, will not fail ere long to fix its age, and assign to it the place it is to occupy in the series of rocks. All our present information respecting the *grande terre* of Guadaloupe amounts to this, that it is a flat limestone country, derived principally from the detritus of zoophytes, with here and there single hills (*mornes*) composed of shell limestone; while Guadaloupe properly so called, separated from the other part by a very narrow channel of the sea, has no trace of limestone, and is entirely volcanic. It is the opinion of father LABAT, who is followed by BUFFON, that the *grande terre*, of a far more recent origin than the other part of Guadaloupe, was originally a shoal covered with corals, which emerged from the deep in consequence of the retreat of the sea. Others, who have visited those parts, such as Messrs. PEYRE, HAPEL, AMIC, &c. find in the irregularity of stratification and the tumultuous manner, in which, as they tell us, the shells are dispersed in the rocks composing the *grande terre*, a strong indication, that this portion of the island

owes its existence to the same subterraneous energies, which still manifest themselves in the eruptions of the *Souffrière*.

M. Lavaisse, to whom I alluded above, as the only author who mentions the galibies, except General Ernouf, speaks of the bed of limestone which encloses them, as the most remarkable of the calcareous rocks in the Leeward islands; I therefore expected to find in his work an exact statement of its mode of occurring; but the only positive information I could collect from this author is, that the bed is a *kilomètre* (nearly an English mile) in length, and that it is covered by the sea at high water. According to him, no trace of shells or organized bodies are discoverable in this rock; but in lieu of these he was fortunate enough to meet with mortars, pestles, hatchets, &c. of a basaltic or porphyritic rock, which, we are informed, were petrified (*petrifiés*). From this very vague account, I should not be induced to lay much stress upon the circumstance that the position of the skeletons is east-west, and that the spot must, therefore, have been a cemetery, which time and circumstances have transformed into a hard calcareous rock.

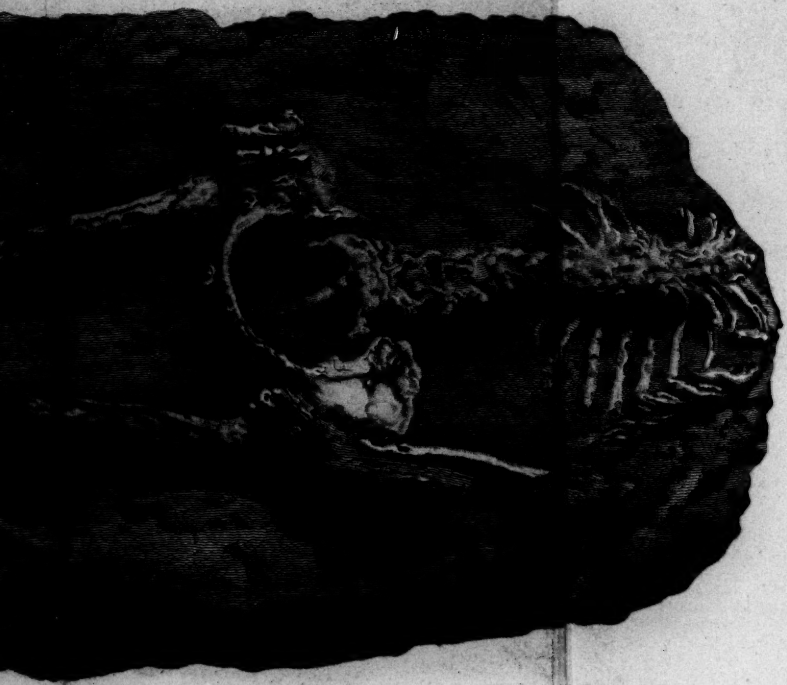
I have to apologize to you, my dear Sir, for this very long letter on a subject which may possibly turn out to be interesting only so far, as the human bones from Guadaloupe are unquestionably the only bones we are acquainted with that have ever been found imbedded in a hard stony mass, that does not appear to belong to common stalactical calcareous depositions. This circumstance admits of being easily ascertained by a close inspection of the locality; and I am perfectly of your opinion, that a comparison of the nature of the different varieties of shell sand with which the neighbourhood of the Caribee islands

abounds, would alone be sufficient to remove many doubts relative to the origin of the bed in question. The sand from thence, which I had an opportunity of seeing, was unlike that of which the stone is composed.

I have the honour to remain, with every sentiment of respect, my dear Sir,

your most obedient
and obliged servant,
CHARLES KONIG.

W. Alexander del.



X. *A new Method of deducing a first Approximation to the Orbit of a Comet from three Geocentric Observations.* *By* James Ivory, *A. M. Communicated by* Henry Brougham, *Esq. F. R. S.*

Read February 17, 1814.

Comets are distinguished from the planets not only by the peculiarities that immediately strike the eye, but likewise by the circumstances attending their motion in the heavens. All the planets move round the sun in orbits nearly circular; they never deviate far from the ecliptic on either side; and they move in a manner not extremely irregular, and in one direction, according to the order of the signs in the zodiac. Comets, on the contrary, when they first come into view, assume gradually greater degrees of brightness, which they again lose by like gradations, and then disappear; thus seeming to visit the neighbourhood of the sun for a short time only, after which they retire into the immensity of space: they are seen in all quarters of the heavens: and their motion is exceedingly various and irregular; confined to no direction; sometimes greatly curved, and often hardly distinguishable from a rectilineal course. If, to phenomena so dissimilar, we add the prejudice which almost universally prevailed, that comets have only a temporary existence, and are produced by occasional causes, we shall not perhaps have much reason to be surprised that the true account of those bodies, which represents them as forming a part of the same system with the planets, eluded the sagacity of Kepler, to whom we are

indebted for the first accurate knowledge of the laws of the planetary motions. This step in our knowledge of the universe was reserved for Sir ISAAC NEWTON. The principal and leading discovery of that great philosopher consisted in generalizing the laws of KEPLER; in proving that they are necessary consequences of a more general fact, namely, that all the planets are continually deflected from a rectilineal motion towards the sun in the inverse proportion of the squares of their distances from that body. He demonstrated that the motions of such a system of bodies must be performed in the conic sections, having the sun in the focus, the species of the curve depending upon the proportion of the rectilineal velocity to the quantity of the deflection towards the common centre. This theory comprehends an infinite variety of motions, all flowing from one common principle; and the ellipse alone, by the changes of form which it undergoes according to the degrees of its eccentricity, seems, at one extreme, when it is greatly elongated, as well adapted to account for the phenomena of the comets, as it is, at the other extreme, when it differs little from a circle, to represent the motions of the planets.

To try if this theory will account for the actual appearances, it is necessary to determine, by means of terrestrial observations, the magnitude and position of the curve in which a comet moves round the sun. Having selected as many observed places of a comet as are necessary for this purpose, the remaining observations will serve as so many tests of the accuracy of the theory. When a comet appears for the first time, it is, indeed, hardly possible to determine its orbit with exactness. The very eccentric ellipses in which those bodies move, allow them to be seen only when they are near the sun; and in this

situation, the nicest observations do not enable us to distinguish the real orbit from the parabola with which it intimately coincides. It is only after one or more reappearances of the same comet, that we can hope to discover the period of its revolution round the sun, and the mean distance of the ellipse in which its motions are really performed: unless indeed a rare instance may sometimes occur, in which the length of time a comet continues visible, and a great number of observations extending over a considerable portion of the orbit, may mark so great deviations from a parabolic motion as to lead to a tolerably exact estimation of the elliptic elements. For a single appearance we must be content with supposing the orbit to be a parabola; a supposition which, if it be not rigorously true, serves important purposes in astronomy: it proves that the comets move round the sun by the same laws as the planets; and it enables us to discover the identity of a comet with one already observed, when we find that they agree in having the same parabolic elements.

Three geocentric observations of the longitude and latitude of a comet are sufficient for determining the parabola which it describes. The problem is one of great difficulty. The apparent motion of a comet is the combined effect of its own motion and of that of the earth; it is therefore extremely irregular and intricate; and on this account it is difficult to deduce the heliocentric positions from observations made on the earth's surface. We can observe the planets at all times and in all situations; and with regard to them we can thus select those positions where the heliocentric places are found immediately from observation, without any perplexed calculations; but we are deprived of this expedient in the case of the comets,

which continue visible for a short time only, and in a small part of their orbit.

In order to evade the difficulties attending a direct consideration of the problem, and to obtain an approximate solution at least, Sir ISAAC NEWTON proposed to take a small portion of the orbit for a straight line described with a uniform motion. On this supposition the projections of the comet on the plane of the ecliptic will lie in one straight line as well as the real places in the heavens; and the several parts of both lines will have the same proportions as the intervals of time between the observations: so that in order to find the projection of a comet's trajectory on the plane of the ecliptic, we have only to draw a straight line which shall cut the several straight lines whose positions are determined by the observed longitudes in such a manner that the intercepted segments shall have given proportions. With three observations only this problem is indeterminate, or admits of innumerable solutions: for, let AP, BE, CQ, (fig. 1. Pl. IV.) drawn in the plane of the ecliptic, represent the directions in which the comet is seen from the earth at the three observations; then, having assumed any point at pleasure, as E, in one of those lines,* we can draw, through that point, a straight line PQ to terminate in the other two lines AP and CQ, so that PE and EQ shall have the same proportion as the intervals of time between the observations. When four observations are employed, the problem, generally speaking, is determinate, and is easily solved.†

* Prin. Math. Lib. 3, Lem. 7.

† Arith. Univ. Prob. 52. Prin. Math. Lib. 1, Lem. 27 Cor. THO. SIMPSON's Elem. Geom. 3d edit. p. 353. DAV. GREG. Astr. Phys. et Geo. Elem. Lib. 5, Prop. 11. Edin. Tran. Vol. III.

If we reflect on the suppositions made by NEWTON for simplifying the method of finding a comet's trajectory, it must be admitted that they seem judicious, and that they are founded in fact. In the usual circumstances in which terrestrial observations are made, there is no doubt that the real path of a comet, for a short time at least, differs little from a straight line, and that the motion is nearly uniform. Yet the problem grounded on these suppositions, although it has been often applied, has never, in any instance, led to a satisfactory determination of a comet's orbit. The reason of this was first noticed by BOSCOVICH, who shewed that in the actual state of the data, owing to the earth and the comet being both in motion, the problem is as indeterminate when four observations are employed, as we have already remarked that it is in the case of three observations only.

In order to explain what has just been said, it must be observed that, for a short time, the earth as well as a comet, may be supposed to move in a straight line with a uniform velocity. If we draw a tangent to that point of the earth's orbit which the planet would occupy at the middle instant between the two extreme observations, the supposition of a uniform motion in the tangent will not much displace the earth from its real positions, and will produce but little alteration in the apparent places of a comet. Suppose then that the earth moves with a uniform motion in the straight line AD, (fig. 2. Pl. IV.) so that the parts AB, BC, CD, are proportional to the intervals of time between four observations of a comet: also let the four observed places of the comet in the ecliptic be represented by the points P, E, F and Q lying in one straight line of which the parts PE, EF, FQ, are proportional to the same intervals of time, and

consequently to the segments AB, BC, CD: then the lines AP, BE, CF, DQ, or the directions in which the comet is seen from the earth, will be given by position on account of the observed longitudes. Draw DM parallel to AP, and PM parallel to AD, and join QM: now if we assume any point whatever in the line AP, as p; and draw pm parallel to PM, and mq parallel to MQ; then, the line pq being drawn, it will be cut by the lines BE and CF into segments having the same proportions as the segments of the lines AD and PQ. To demonstrate this: draw BH and CG parallel to AP and DM, and let those lines meet PM in H, G, and pm in h, g: join HE, GF; and because the segments of PM are equal to the segments of AD, they will be proportional to the segments of PQ (hyp.); therefore (E. 2. VI.) HE and GF will be parallel to MQ and mq: draw he and gf parallel to HE and GF; and consequently (E. 2. VI.) pq will be cut in e and f in the same proportion as AD is cut in B and C, and PQ in H and G: and I say that BE will cut pq in e, and CF will cut it in f. For on account of the parallel lines, and because PM = pm, PG = pg, PH = ph, therefore

$$QM : HE :: qm : he$$
$$QM : GF :: qm : gf$$

and, by alternation,

$$QM : qm :: HE : he$$
$$QM : qm :: GF : gf$$

but, on account of the similar triangles QDM, qdm,

$$QM : qm :: MD : mD$$

therefore, because MD = GC = BH, and $mD = Cg = Bh$,

$$BH : Bh :: HE : he$$
$$CG : Cg :: GF : gf$$

consequently BE will cut *pq* in *e*, and CF will cut it in *f*. Thus it is proved that innumerable lines may be drawn that will all be cut by the four lines AP, BE, CF, DQ into segments having the same proportions as the intervals of time between four observations of a comet. The problem is therefore not more determinate with four, or even a greater number of, observations, than it is with three: and it is totally unfit for finding the lengths of the lines AP, BE, CF, DQ, or the comet's distances from the earth. The shorter we make the intervals of time between the observations, and the more exactly the suppositions made by Sir ISAAC NEWTON are fulfilled, the less fitted is the problem to answer the purposes of astronomy, because it approaches more nearly to the conditions which make it indeterminate.*

Besides the method of which we have hitherto been speaking, Sir ISAAC NEWTON likewise gave another very able solution of the problem,† which however is too tedious and laborious in practice to fulfil the wishes of astronomers, and it is now disused. Dr. HALLEY, who studied the astronomy of comets with much diligence, has no where explained the method he followed in determining their orbits. Since his time this problem has not been much discussed in England; but it is the subject of a great number of researches scattered in the academical collections of the learned societies on the continent. All the resources of the modern mathematics have been directed to overcome the peculiar difficulties of this intricate investigation. Yet it must be confessed that practical astronomy has not reaped so much benefit as might justly

* Vide a Memoir by Professor PLAYFAIR in the 3d Vol. of the Edin. Trans.

† Prin. Math. Lib. 3, Prop. 41.

have been expected from the labours of so many eminent men. In most of the solutions that have been proposed we find certain coefficients which are not only extremely small when reduced into numbers, but they likewise suffer great alterations in their relative magnitudes when small changes, within the limits of the probable errors, are made in the quantities furnished by observation. In such circumstances the results obtained are often wide of the truth, and in no case can entire dependence be had on their accuracy. The fact is that the solutions here spoken of, are uncertain and useless in practice for the very same reason as the one already mentioned, which supposes the orbit to be rectilineal, and proposes to deduce it from four observations. Nor is this to be wondered at. We have already shewn that it is not any inaccuracy in the suppositions made by Sir ISAAC NEWTON, which renders his first method insufficient and of no use in practical astronomy; nor is this owing to any defectiveness in the geometrical construction; it arises from his having overlooked that connection between the motion of the earth and the motion of the comet, which makes the data approach so near the indeterminate case of the problem that the conclusion becomes quite uncertain. We fall upon the indeterminate case when we suppose both the earth and the comet to move in straight lines with uniform velocities; and the very same hypothesis will be found to make the small coefficients alluded to, accurately evanescent.* The inconvenience is inherent in the quantities obtained by

* The small coefficients here mentioned, are exactly evanescent when all the three observed places of a comet are in one great circle of the heavens: but it is easy to prove that if two celestial bodies be supposed to move uniformly in straight lines, one of them will be seen from the other to describe constantly the same great circle.

observation, and it cannot be entirely obviated merely by pursuing new methods of solution, nor by pushing calculation to greater degrees of exactness. The problem cannot be justly and usefully solved, unless such coefficients are excluded, and the result is obtained by means of quantities on which the errors of observation have no more influence than they ought to have.

There are however some solutions of this problem to which the preceding observations must not be applied. Of this kind is the method of M. Boscovich; that of the celebrated Laplace;* and those which Legendre has more lately published:† all of which have been found useful in practical astronomy. The method of Boscovich owes its utility as an approximation to the circumstance of introducing the velocity in the orbit as a principal condition: for that velocity depending upon the proportion of the distances of the earth and the comet from the sun, limits the other conditions, and places the orbit in its proper situation. The same thing may be said of the methods of Laplace and Legendre: and, in general, we may affirm that no solution of this problem can be free from the imperfections we have pointed out, in which the velocity in the orbit, or some equivalent property, does not enter as a principal condition.

In order to place what has been said in a clearer light, it is to be observed that three complete observations of a celestial body are sufficient for determining the species, the magnitude, and the position of the curve in which it moves round the sun. On this account there is a superfluity of conditions when we

* Méc. Céleste, Tom. 1. Liv. 2, chap. 4.

† Nouvelles Méthodes pour la Détermination des Orbites des Cometes, 1806.

suppose the orbit to be a parabola: because, in this case, the velocity in the orbit furnishes an equation without introducing any new unknown quantity. Thus it happens, that in the problem of the comets there is one equation more than there are quantities sought: and by combining those equations in different ways, various solutions of the problem may be obtained. But it ought likewise to be observed, that if we set aside the equation derived from the nature of the orbit, the remaining ones, in the actual state of the data, will nearly coincide with what would result from the hypothesis of a uniform motion in a straight line: and although, theoretically speaking, we can solve the problem by means of those equations, yet we shall thus infallibly introduce coefficients that are small and ill defined, and unfit for any practical purpose. It is therefore necessary to include the velocity in the orbit, or some equivalent property, if we wish to obtain a solution useful in practical astronomy: and even when this mode of solution is adopted, it is still necessary to examine with care the quantities introduced by combining the other conditions, in order to exclude the faulty coefficients we have been speaking of.

It is of the greater consequence to discuss the peculiarities of this problem, because the observations of comets are susceptible of little accuracy even with the best instruments and the greatest care, on account of the haze, or coma, with which those bodies are generally surrounded. Every solution of a physical problem which is deduced from quantities that are of the same order as the unavoidable errors in the data furnished by observation, can plainly be of no practical utility; and it is in this predicament that those coefficients stand, which would

vanish exactly on the supposition that the earth and a comet moved, for a short time, with uniform velocities in straight lines.

On the whole it appears from what has been said, that the same circumstance to which is owing the failure of Sir ISAAC NEWTON's first method of finding a comet's trajectory, has produced like effects in many of the later solutions of this problem. This is indeed no more than what was to be expected: and it must excite surprise that more attention was not paid to the peculiarities of this investigation, after BOSCOVICH had fully developed the solution of NEWTON, and shown the cause of its want of success.

The English astronomer will find the methods of BOSCOVICH and LAPLACE, properly illustrated by examples of all the computations, in a work published in 1793, by Sir HENRY ENGLEFIELD; in which the author has judiciously selected all that is practically useful from the numerous writings on this subject at that time before the public.

The method of LAPLACE would be rigorous, were it possible to find exactly the numerical quantities that enter into his formulas. These are the first and second differential coefficients of the longitude and latitude considered as functions of the time; the values of which we can do nothing more than determine nearly by interpolating the observed places of a comet. With three observations only, the quantities thus found cannot be expected to have much accuracy, more especially if the motion in longitude, or latitude, be quick and irregular, as often happens: and when a greater number of observations is employed, there arises a new cause of inaccuracy in the arithmetical operations necessary for interpolating, which augment

the errors of observation, always considerable of themselves in the case of the comets. These remarks, which are certainly just, are made by LEGENDRE in his memoir on this problem, published in 1806. That geometer therefore thinks it preferable to deduce the orbit immediately from the observed places of a comet: and we agree with his opinion, that the observations will be better represented this way, than by employing the method of LAPLACE. With regard to the solutions of the problem which LEGENDRE himself has given, it will be admitted that they approximate sufficiently near to the elements of the orbit, to answer all the purposes of practical astronomy; but his formulas are complicated; and the number of equations which it is necessary to form and to solve, seems to render his methods ill adapted for general use. It is the object of this paper to give a new solution of this problem, which, while it does not yield to any of the known methods in accuracy of result, I judge, will be found as commodious in practice as the nature of such a calculation can well admit.

1. Let the coordinates that determine the position of a comet with regard to the ecliptic, and a straight line drawn through the first point of Aries, (which point is supposed to be immoveable) be represented by x, y, z; of which the last is perpendicular to the plane of the ecliptic, and the other two have their origin in the sun's centre: and likewise let the comet's distance from the sun, or the radius vector of the orbit, be represented by r. Farther, supposing the mean distance of the earth's orbit to be unit, put m for the length of the circular arc of the mean motion described in a second of time; then $\frac{m^2}{4}$, the versed-sine of that arc, will be the space

through which the earth descends towards the sun in a second of time; and m^2 is the velocity produced by the solar force; therefore the direct velocity towards the sun produced by the same force acting on the comet at the distance r, is $\frac{m^2}{r^2}$; and, this being decomposed into the equivalent velocities parallel to the coordinates, these velocities will be $\frac{m^2x}{r^3}$, $\frac{m^2y}{r^3}$, $\frac{m^2z}{r^3}$ respectively, which are the actual diminutions of the velocities with which the coordinates increase; but if $d\tau$ represent the fluxion of the time, the diminutions of the velocities with which the coordinates increase, will be represented by $-\frac{ddx}{d\tau^2}$, $-\frac{ddy}{d\tau^2}$, $-\frac{ddz}{d\tau^2}$; therefore we shall have these equations, viz.

$$\frac{m^2x}{r^3} = -\frac{ddx}{d\tau^2};\ \frac{m^2y}{r^3} = -\frac{ddy}{d\tau^2};\ \frac{m^2z}{r^3} = -\frac{ddz}{d\tau^2}.$$

Now if we put τ to denote the mean motion of the earth, proportional to the time, and estimated by the arc of the circle whose radius is the mean distance, or unit; then $d\tau^2$ will denote the same thing that is signified by m^2dt^2 in the foregoing equations, which will thus become,

$$\begin{aligned} \frac{ddx}{d\tau^2} + \frac{x}{r^3} &= 0 \\ \frac{ddy}{d\tau^2} + \frac{y}{r^3} &= 0 \qquad (1) \\ \frac{ddz}{d\tau^2} + \frac{z}{r^3} &= 0. \end{aligned}$$

Multiply these equations by $2dx$, $2dy$, $2dz$, respectively; then, because $2xdx + 2ydy + 2zdz = 2rdr$, we shall get by adding them

$$\frac{2dxddx + 2dyddy + 2dzddz}{d\tau^2} + \frac{2dr}{r^2} = 0;$$

and, by integrating,

$$\frac{dx^2 + dy^2 + dz^2}{d\tau^2} - \frac{2}{r} = \text{const.}$$

To determine the constant quantity in this equation, it is to be observed that the first term is the square of the velocity in the orbit: if then we suppose the orbit to be an ellipse of which f is the mean distance; $f.\varepsilon$ the eccentricity; and v and v' the velocities at the perihelion and aphelion distances; the foregoing equation will become at those two points of the orbit,

$$v^2 - \frac{2}{f(1-\varepsilon)} = \text{const.}$$

$$v'^2 - \frac{2}{f(1+\varepsilon)} = \text{const.}$$

and if we multiply these by $(1-\varepsilon)^2$ and $(1+\varepsilon)^2$ respectively, and then subtract them, we shall get

$$v^2(1-\varepsilon)^2 - v'^2.(1+\varepsilon)^2 + \frac{4\varepsilon}{f} = -4\varepsilon \times \text{const.}$$

but $v.f(1-\varepsilon) - v'.f(1+\varepsilon) = 0$; for these quantities are respectively the doubles of the sectors described at the perihelion and aphelion in the time denoted by unit: hence $-\frac{1}{f}$ = const.: the foregoing equation will therefore become,

$$\frac{dx^2+dy^2+dz^2}{dt^2} - \frac{2}{r} + \frac{1}{f} = 0.$$

Again let the equations (1), multiplied by x, y, z respectively, be added to the last equation; then observing that $xddx + yddy + zddz + dx^2 + dy^2 + dz^2 = \frac{dd.r^2}{2}$, we shall get

$$\frac{1}{2}.\frac{dd.r^2}{dt^2} - \frac{1}{r} + \frac{1}{f} = 0.$$

Let r^0 denote the value of r, and $2p$ that of $\frac{d.r^2}{dt}$, at the epoch from which the time is reckoned; and further let the value of r^2 be assumed in a series with indeterminate coefficients, as follows, viz.

$$r^2 = r^{02} + 2pt + At^3 + Bt^5 \text{ \&c.:}$$

then $\frac{1}{r} = \frac{1}{r^0} - \frac{pr}{r^3}$ &c.; and if these values be substituted in the foregoing equation the coefficients A, B, &c. will be readily determined; which being done, we shall get

$$r^2 = r^{02} + 2pr + \left(1 - \frac{r^0}{f}\right) \cdot \frac{\tau^2}{r^0} - \frac{p}{3} \cdot \frac{\tau^3}{r^{03}} \text{ &c.}$$

It is next necessary to integrate the equations (1). I begin with seeking an incomplete integral of this form, viz.

$$s = 1 + C\tau^2 + D\tau^3 \text{ &c.:}$$

and, having substituted in the first of the equations (1), s for x; and, for $\frac{1}{r^3}$, its value taken from the preceding series for r^2; I get,

$$\frac{dds}{d\tau^2} + \frac{s}{r^{03}} \cdot \left\{1 - \frac{3pr}{r^0} \text{ &c.}\right\} = 0$$

by means of which equation, the coefficients in the series for s, are easily found: then

$$s = 1 - \frac{1}{2} \cdot \frac{\tau^2}{r^{03}} + \frac{p}{2} \cdot \frac{\tau^3}{r^{03}} \text{ &c.} \quad (2).$$

I now seek another incomplete integral of this form, viz.

$$\sigma = \tau + E \cdot \tau^3 + F \cdot \tau^4 \text{ &c.:}$$

and, having substituted σ for x, I get,

$$\frac{dd\sigma}{d\tau^2} + \frac{\sigma}{r^{03}} \cdot \left\{1 - \frac{3pr}{r^0} \text{ &c.}\right\} = 0;$$

whence the coefficients of the series for σ will be readily found; then

$$\sigma = \tau - \frac{1}{6} \cdot \frac{\tau^3}{r^{03}} + \frac{p}{4} \cdot \frac{\tau^4}{r^{03}} \text{ &c.} \quad (3).$$

The complete integrals of the equations (1) can now be obtained, and they are

$$x = sx^0 + \sigma \frac{dx}{d\tau}$$

$$y = sy^0 + \sigma \frac{dy}{d\tau}$$

$$z = sz^0 + \sigma \frac{dz}{d\tau}:$$

where x° and $\frac{dx^\circ}{d\tau}$, which denote what x and $\frac{dx}{d\tau}$ become at the epoch, are the arbitrary quantities necessary to complete the fluent: and the same things are to be understood in the expressions of y and z.

2. Of three observations which are necessary to determine a comet's orbit, let the epoch, or the instant from which the time is reckoned, be made to fall on the middle one; and let $-\tau$ denote the interval between the first and second observations, which precedes the epoch; and $+\tau'$, the interval between the second and third observations, which follows the epoch; and further let x, y, z and r express the coordinates and radius vector of the comet at the first observation; x°, y°, z°, and r°, the same quantities at the second observation; and x', y', z', and r', those at the third: finally, let the values of the series denoted by s and σ (Equat. 2 and 3) be taken for the intervals $-\tau$ and $+\tau'$; that is, let

$$s = 1 - \tfrac{1}{2} \cdot \frac{r^\circ}{r^{\circ 3}} - \frac{p r^3}{2 r^{\circ 3}}$$

$$-\sigma = -\tau + \frac{r^\circ}{6 r^{\circ 3}} + \frac{p r^\circ}{4 r^{\circ 3}}$$

$$s' = 1 - \tfrac{1}{2} \cdot \frac{r'^3}{r^{\circ 3}} + \frac{p r'^3}{2 r^{\circ 3}}$$

$$\sigma' = \tau' - \tfrac{1}{6} \cdot \frac{r'^3}{r^{\circ 3}} + \frac{p r'^3}{4 r^{\circ 3}};$$

then, on account of the equations (4), we shall get,

$$x = s x^\circ - \sigma \frac{dx^\circ}{d\tau}$$

$$y = s y^\circ - \sigma \frac{dy^\circ}{d\tau}$$

$$z = s z^\circ - \sigma \frac{dz^\circ}{d\tau}$$

$$x' = s' x^\circ + \sigma' \frac{dx^\circ}{d\tau}$$

$$y' = s'y^\circ + \sigma' \frac{dy^\circ}{d\tau}$$

$$z' = s'z^\circ + \sigma' \frac{dz^\circ}{d\tau}$$

and if these equations be combined two and two so as to exterminate $\frac{dx^\circ}{d\tau}$, $\frac{dy^\circ}{d\tau}$, $\frac{dz^\circ}{d\tau}$, we shall get

$$\begin{aligned} \sigma' x + \sigma x' &= (\sigma' s + \sigma s')\, x^\circ \\ \sigma' y + \sigma y' &= (\sigma' s + \sigma s')\, y^\circ \qquad (5) \\ \sigma' z + \sigma z' &= (\sigma' s + \sigma s')\, z^\circ : \end{aligned}$$

Finally let σ'' be written for $(\sigma' s + \sigma s')$; then, by substituting the series that σ, σ', s, s' stand for, we shall get

$$\sigma'' = (\tau + \tau') - \tfrac{(\tau+\tau')^3}{6r^3} - \tfrac{p}{4} \cdot \tfrac{(\tau-\tau')(\tau+\tau)^2}{r^3} \text{ \&c.} \quad (6).$$

3. Employing the same marks as in the case of the coordinates, to distinguish the known quantities of each observation, I shall write,

c, c°, c'; for the three geocentric longitudes of a comet:

λ, λ°, λ'; for the three geocentric latitudes:

ρ, ρ°, ρ'; for the comet's distances from the earth:

e, e°, e'; for the earth's longitudes:

R, R°, R′; for the earth's distances from the sun:

by means of which quantities, the coordinates of the comet at the first observation will be thus expressed, viz.

$$\begin{aligned} x &= \mathrm{R} \cos. e + \rho \cos. \lambda \cos. c \\ y &= \mathrm{R} \sin. e + \rho \cos. \lambda \sin. c \\ z &= \rho \sin. \lambda : \end{aligned}$$

and in like manner the expressions of the coordinates at the second and third observations will be formed by placing the characteristic marks on the several letters. By substituting the values thus obtained in the equations (5), we shall get these other equations, viz.

$$\sigma'\rho\cos.\lambda\cos.c + \sigma\rho'\cos.\lambda'\cos.c' - \sigma''\rho^{\circ}\cos.\lambda^{\circ}\cos.c^{\circ} =$$
$$-\sigma'R\cos.e - \sigma R'\cos.e' + \sigma''R^{\circ}\cos.e^{\circ}$$
$$\sigma''\rho\cos.\lambda\sin.c + \sigma\rho'\cos.\lambda'\sin.c' + \sigma''\rho^{\circ}\cos.\lambda^{\circ}\sin.c^{\circ} = \quad (7)$$
$$-\sigma'R\sin.e - \sigma R'\sin.e' + \sigma''R^{\circ}\sin.e^{\circ}$$
$$\sigma'\rho\sin.\lambda + \sigma\rho'\sin.\lambda' - \sigma''\rho^{\circ}\sin.\lambda^{\circ} = o.$$

These equations are universally true of all orbits; and, in order to solve the general problem of finding a planet's orbit, they require only to be properly discussed with the view of obtaining the values of $\rho, \rho^{\circ}, \rho'$ by means of formulas sufficiently exact and commodious in practice: but, at present, I confine my attention to the case of the comets moving in parabolic orbits.

4. We may apply the equations (5), which are general for all orbits, to the orbit of the earth: let μ, μ', μ'' represent what σ, σ', σ'' become in the case of that orbit; that is (neglecting quantities of the fourth and higher orders), let (Equat. 2, 3, 6)

$$\mu = \tau - \frac{\tau^3}{6R^3}$$
$$\mu' = \tau' - \frac{\tau'^3}{6R^3}$$
$$\mu'' = (\tau + \tau') - \frac{(\tau+\tau')^3}{6R^3};$$

then, Equat. (8),

$$\mu'R\cos.e + \mu R'\cos.e' - \mu''R^{\circ}\cos.e^{\circ} = o$$
$$\mu'R\sin.e + \mu R\sin.e' - \mu''R^{\circ}\sin.e^{\circ} = o.$$

On account of these formulas, we have

$$\sigma'R\cos.e + \sigma R'\cos.e' - \sigma''R^{\circ}\cos.e^{\circ} =$$
$$(\sigma' - \mu')R\cos.e + (\tau - \mu)R\cos.e' - (\sigma'' - \mu'')R^{\circ}\cos.e^{\circ};$$

and, by substituting the series that σ, σ', σ''; μ, μ', μ''; stand for, in the right hand side of this equation, and neglecting quantities of the fourth and higher orders, we shall get

$$\sigma''R\cos.e + \sigma R'\cos.e' - \sigma''R^\circ\cos.e^\circ =$$
$$\frac{1}{6}\cdot\left(\frac{1}{\rho^3}-\frac{1}{R^{\circ 3}}\right)\cdot\left\{-\tau'^2R\cos.e-\tau^2R'\cos.e'+(\tau+\tau')^2R^\circ\cos.e^\circ\right\}$$

or, which is the same thing,

$$\sigma''R\cos.e+\sigma R'\cos.e'-\sigma''R^\circ\cos.e^\circ=\frac{\tau\tau'(\tau+\tau')}{2}\cdot\left(\frac{1}{\rho^3}-\frac{1}{R^{\circ 3}}\right).R^\circ\cos.e^\circ$$
$$-\frac{\tau'^2}{6}\cdot\left(\frac{1}{\rho^3}-\frac{1}{R^{\circ 3}}\right).(R\cos.e-R^\circ\cos.e^\circ)$$
$$-\frac{\tau^2}{6}\cdot\left(\frac{1}{\rho^3}-\frac{1}{R^{\circ 3}}\right).(R'\cos.e'-R^\circ\cos,e^\circ);$$

but $(R\cos.e - R^\circ\cos.e^\circ)$ and $(R'\cos.e' - R^\circ\cos.e^\circ)$ are quantities of the first order; consequently the two terms that contain them, will be of the fourth order, and may therefore be rejected: thus we get

$$\sigma''R\cos.e+\sigma R'\cos.e'-\sigma''R^\circ\cos.e^\circ=\frac{\tau\tau'(\tau+\tau')}{2}\cdot\left(\frac{1}{\rho^3}-\frac{1}{R^{\circ 3}}\right)\cdot R^\circ\cos.e^\circ.$$

And, in the very same manner it may be shewn that

$$\sigma''R\sin.e+\sigma R'\sin.e'-\sigma''R^\circ\sin.e^\circ=\frac{\tau\tau'(\tau+\tau')}{2}\cdot\left(\frac{1}{\rho^3}-\frac{1}{R^{\circ 3}}\right)\cdot R^\circ\sin.e^\circ.$$

Let the values just obtained be substituted in the two first of the equations (7), and they will become

$$\sigma''\rho\cos.\lambda\cos.c+\sigma\rho'\cos.\lambda'\cos.c'-\sigma''\rho^\circ\cos.\lambda^\circ\cos.c^\circ=$$
$$-\frac{\tau\tau'(\tau+\tau')}{2}\cdot\left(\frac{1}{\rho^3}-\frac{1}{R^{\circ 3}}\right).R^\circ\cos.e^\circ$$
$$\sigma''\rho\cos.\lambda\sin.c+\sigma\rho'\cos.\lambda'\sin.c'-\sigma''\rho^\circ\cos.\lambda^\circ\sin.c^\circ=$$
$$-\frac{\tau\tau'(\tau+\tau')}{2}\cdot\left(\frac{1}{\rho^3}-\frac{1}{R^{\circ 3}}\right).R^\circ\sin.e^\circ:$$

Further let n denote an angle, to be afterwards determined; then, by combining the two last equations we shall readily obtain,

$$\sigma''\rho\cos.\lambda\cos.(c-n)+\sigma\rho'\cos.\lambda'\cos.(c'-n)-\sigma''\rho^\circ\cos.\lambda^\circ$$
$$\times\cos.(c^\circ-n)=-\frac{\tau\tau'(\tau+\tau')}{2}\cdot\left(\frac{1}{\rho^3}-\frac{1}{R^{\circ 3}}\right).R^\circ\cos.(e^\circ-n)$$

$\sigma'\rho \cos.\lambda \sin.(c-n) + \sigma\rho' \cos.\lambda' \sin.(c'-n) - \sigma''\rho^{\circ} \cos.\lambda^{\circ} \times \sin.(c^{\circ}-n) = -\frac{\pi'(\tau+\tau')}{2} \cdot \left(\frac{1}{r^{03}} - \frac{1}{R^{03}}\right) \cdot R^{\circ} \sin.(e^{\circ}-n).$

Conceive a great circle to be drawn on the surface of the sphere through the geocentric places of the comet at the two extreme observations; and let n, in the foregoing equations, denote the longitude of one of the intersections of that great circle with the ecliptic: also let h and h' denote the arcs of the great circle intercepted between the same intersection and the places of the comet abovementioned; then these arcs will be the hypothenuses of two right-angled triangles of which the sides, perpendicular to the ecliptic, are the arcs λ and λ'; and the other sides, in the ecliptic, are the arcs $(c-n)$ and $(c'-n)$: further, let i denote the inclination of the same great circle to the ecliptic; then will i be an angle common to the two right-angled triangles abovementioned, opposite to the sides λ and λ'; and, by the properties demonstrated in spherical trigonometry, we shall get these formulas, viz.

$$\begin{aligned}
\cos.\lambda \cos.(c-n) &= \cos. h \\
\cos.\lambda' \cos.(c'-n) &= \cos. h' \\
\cos.\lambda \sin.(c-n) &= \cos. i \sin. h \\
\cos.\lambda' \sin.(c'-n) &= \cos. i \sin. h' \\
\sin.\lambda &= \sin. i \sin. h \\
\sin.\lambda' &= \sin. i \sin. h'.
\end{aligned}$$

Now let these values be substituted in the two foregoing equations, and likewise in the last of the equations (7), and we shall get,

$$\sigma'\rho \cos. h + \sigma\rho' \cos. h' - \sigma''\rho^{\circ} \cos.\lambda^{\circ} \cos.(c^{\circ}-n) = -\frac{\pi'(\tau+\tau')}{2} \cdot \left(\frac{1}{r^{03}} - \frac{1}{R^{03}}\right) \cdot R^{\circ} \cos.(e^{\circ}-n) \qquad (8)$$

$$\sigma'\rho\cos.i\sin.h+\sigma\rho'\cos.i\sin.h'-\sigma''\rho^\circ\cos.\lambda^\circ\sin.(\epsilon^\circ-n)=$$
$$-\frac{\tau\tau'(\tau+\tau')}{2}\cdot\left(\frac{1}{r^{\circ 3}}-\frac{1}{R^{\circ 3}}\right)\cdot R^\circ\sin.(\epsilon^\circ-n) \quad (8)$$

$\sigma'\rho\sin.i\sin.h+\sigma\rho'\sin.i\sin.h'-\sigma''\rho^\circ\sin.\lambda^\circ=0$:
and finally if we subtract the two last of these equations after having divided them by cos. i and sin. i, we shall get

$$\sigma''\rho^\circ\left\{\frac{\cos.\lambda^\circ\sin.(\epsilon^\circ-n)}{\cos.i}-\frac{\sin.\lambda^\circ}{\sin.i}\right\}=\frac{(\tau+\tau')\tau\tau'}{2}\cdot\left(\frac{1}{r^{\circ 3}}-\frac{1}{R^{\circ 3}}\right)\cdot\frac{R^\circ\sin.(\epsilon^\circ-n)}{\cos.i} \quad (9).$$

5. In the last equation, all the terms being of the third order, we may consider σ'' as equal to $(\tau+\tau')$: therefore, by division, we shall get

$$\rho^\circ\times\frac{\cos.\lambda^\circ}{\sin.(\epsilon^\circ-n)}\cdot\left\{\sin.(\epsilon^\circ-n)-\frac{\tan.\lambda^\circ}{\tan.i}\right\}=\frac{\tau\tau'}{2}\cdot\left(\frac{1}{r^{\circ 3}}-\frac{1}{R^{\circ 3}}\right)\cdot R^\circ;$$

or, by introducing a new letter, we shall have these two formulas, equivalent to the last one, viz.

$$\zeta=\frac{\cos.\lambda^\circ}{\sin.(\epsilon^\circ-n)}\cdot\left\{\sin.(\epsilon^\circ-n)-\frac{\tan.\lambda^\circ}{\tan.i}\right\}$$
$$\zeta\times\rho^\circ=\frac{\tau\tau'}{2}\cdot\left(\frac{1}{r^{\circ 3}}-\frac{1}{R^{\circ 3}}\right)\cdot R^\circ. \quad (10)$$

If we suppose all the three geocentric places of the comet to be situated in the same great circle of the heavens; then sin. $(\epsilon^\circ-n)$. tan. $i=$ tan. λ°: in this case therefore $\zeta=0$, which requires that $\frac{1}{r^{\circ 3}}-\frac{1}{R^{\circ 3}}=0$, and $r^\circ=R^\circ$. Thus we learn that a comet and the earth are equally distant from the sun, when the comet's apparent motion continues for a short time to be performed in one great circle of the heavens. Again if ζ be positive, then $\left(\frac{1}{r^{\circ 3}}-\frac{1}{R^{\circ 3}}\right)$ will be positive too, and r° will be less than R°: but if ζ be negative, then $\left(\frac{1}{r^{\circ 3}}-\frac{1}{R^{\circ 3}}\right)$ will likewise be negative, and r° will be greater than R°. But there are two cases when ζ will be indeterminate, and the preceding rules

will fail. This will happen when the comet moves exactly in the plane of the ecliptic, in which case $\frac{\tan.\lambda^{\circ}}{\tan.i}$ is indeterminate. The same thing will likewise happen when the great circle which passes through the two extreme places of the comet cuts the ecliptic in the points occupied by the earth and the sun at the middle observation: for in this case, sin. $(c^{\circ}-n)=0$; now since ζ cannot become infinitely great, it is necessary that $\left\{\text{sin. }(c^{\circ}-n)-\frac{\tan.\lambda^{\circ}}{\tan.i}\right\}$ vanish together with sin. $(c^{\circ}-n)$, which will make ζ indeterminate.

The formula for ζ, which by its sign alone enables us to judge of the magnitude of r°, contains the angles n and i: and it is necessary to investigate rules for computing those quantities. By attending to what n and i were made to signify it will readily appear that

$$\text{tan. } i=\frac{\text{tan. }\lambda}{\text{sin. }(c-n)}=\frac{\text{tan. }\lambda'}{\text{sin. }(c'-n)};$$

let $f=\frac{\text{tan. }\lambda'}{\text{tan. }\lambda}$; then

$$f\text{ sin. }(c-n)=\text{sin. }(c'-n):$$

but

$$\text{sin. }(c-n)=\text{sin. }\left(\tfrac{c'+c}{2}-n\right)\text{cos. }\tfrac{c'-c}{2}-\text{cos. }\left(\tfrac{c'+c}{2}-n\right)\text{sin. }\tfrac{c'-c}{2}$$

$$\text{sin. }(c'-n)=\text{sin. }\left(\tfrac{c'+c}{2}-n\right)\text{cos. }\tfrac{c'-c}{2}+\text{cos. }\left(\tfrac{c'+c}{2}-n\right)\text{sin. }\tfrac{c'-c}{2};$$

therefore by substitution and division we shall get

$$\text{tan. }\left(\tfrac{c'+c}{2}-n\right)=\frac{f+1}{f-1}\text{. tan. }\tfrac{c'-c}{2}:$$

and, if $f=\frac{\text{tan. }\lambda'}{\text{tan. }\lambda}=\text{tan. }(45^{\circ}\pm\varphi)$; then

$$\text{tan. }\left(\tfrac{c'+c}{2}-n\right)=\pm\frac{\text{tan. }\frac{c'-c}{2}}{\text{tan. }\varphi}=\pm\text{ tan. }\tfrac{c'-c}{2}\text{ cot. }\varphi.$$

To the value of tan. $\left(\frac{c'+c}{2}-n\right)$, found by this rule, there

will correspond two arcs, differing from one another by 180°, which will determine both the points of intersection of the great circle and the ecliptic; but it will be sufficient to take that one which immediately precedes the comet in longitude.

The arc n being thus found, we have these formulas for finding i, viz.

$$\text{tan. } i = \frac{\text{tan. } \lambda}{\text{sin. } (c-n)} = \frac{\text{tan. } \lambda'}{\text{sin. } (c-n)};$$

and the double value will serve to prove the accuracy of the calculation.

But we may determine whether r° is greater or less than R°, without any calculation. Let h° denote the arc of a great circle drawn from the intersection whose longitude is n, to the geocentric place of the comet at the middle observation: then h° will be the hypothenuse of a right-angled triangle, having the arc $(c^\circ - n)$ of the ecliptic for one side, and λ°, perpendicular to the ecliptic, for the other side: and if we put i° to denote the angle of the triangle opposite to λ°, we shall have

$$\text{cos. } \lambda^\circ \text{ sin. } (c^\circ - n) = \text{cos. } i^\circ \text{ sin. } h^\circ$$

$$\text{sin. } \lambda^\circ = \text{sin. } i^\circ \text{ sin. } h^\circ.$$

Substitute these values in the equation (9), and we shall get, by division,

$$\rho^\circ \cdot \frac{\text{sin. } (i - i^\circ) \cdot \text{sin. } b^\circ}{\text{sin. } i \text{ sin. } (c^\circ - n)} = \frac{\tau\tau'}{2} \cdot \left(\frac{1}{r^3} - \frac{1}{R^3}\right) \cdot R^\circ.$$

Conceive two arcs to be drawn perpendicular to the great circle which passes through the geocentric places of the comet at the two extreme observations; one, denoted by π, drawn from the extremity of h°, that is, from the geocentric place of the comet at the middle observation; and the other denoted by Π, drawn from the place in the ecliptic occupied by the earth at the same time: then, observing that π will be one side of a right-angled spherical triangle, of which h° is the

hypothenuse, and $(i - i^\circ)$ the angle opposite to ϖ; it is plain that we shall have

$$\text{sin. } \varpi = \text{sin. } (i - i^\circ) \text{ sin. } h^\circ$$
$$\text{sin. } \Pi = \text{sin. } i \text{ sin. } (e^\circ - n).$$

Therefore, by substitution, the last equation will become

$$\rho^\circ \times \frac{\text{sin. } \varpi}{\text{sin. } \Pi} = \frac{\pi'}{2} \cdot \left(\frac{1}{r^3} - \frac{1}{R^3}\right) \cdot R^\circ.$$

This equation is equivalent to the second of the equations (10): and we infer from it that r° will be less, or greater, than R°, according as $\frac{\text{sin. } \varpi}{\text{sin. } \Pi}$ is positive or negative; that is, according as the arcs ϖ and Π are on the same side, or on different sides, of the great circle to which they are both perpendicular. Hence if we mark, on the surface of a celestial globe, three geocentric places of a comet, and likewise the places in the ecliptic occupied by the earth at the middle observation; then the comet's distance from the sun will be greater than the earth's distance, when the great circle drawn through the two extreme places of the comet passes between the earth's place, and the remaining place of the comet; but when these two places are both on the same side of the great circle, the comet's distance from the sun will be less than the earth's distance. When all the three places of the comet are in one great circle of the heavens, then $\varpi = 0$, and $\frac{\text{sin. } \varpi}{\text{sin. } \Pi} = 0$: and in this case, the comet and the earth will be equally distant from the sun. From these rules we must however except the cases in which $\text{sin. } \Pi = 0$: for, since $\frac{\text{sin. } \varpi}{\text{sin. } \Pi}$ cannot become infinite, it is necessary that the numerator vanish together with the denominator, so that $\frac{\text{sin. } \varpi}{\text{sin. } \Pi}$ will become indeterminate. This will happen, as

before observed, when the comet's orbit coincides with the ecliptic, in which case sin. $i = 0$; and likewise when the great circle drawn through the two extreme geocentric places of the comet cuts the ecliptic in the places of the earth and the sun at the middle observation, in which case sin. $(\epsilon^{\circ} - n) = 0$.

The inferences here drawn from the preceding analysis coincide with the rules first given by M. LAMBERT, of Berlin, for judging of a comet's distance from the sun by the inflection of its apparent path.

6. The second of the equations (10) contains only one unknown quantity, namely ρ°: and hence it may be thought that we have already, by means of that equation, obtained a solution of the problem, which is both simple and elegant. And this would undoubtedly be the case, were it not that the coefficient ζ is always extremely small and greatly affected with the errors of observation. It depends entirely on the deviation of the comet's apparent path from a great circle of the heavens; and this deviation is often so little, that small changes in the observed places of the comet, by no means inconsistent with the errors of observation, will make ζ evanescent, or even take a different sign from what it had before. If we suppose the motions both of the earth and the comet to be rectilineal and uniform, which is never far from the truth in the short intervals that must intervene between the observations selected for finding a comet's orbit; then the apparent motion of the comet would be accurately in a great circle of the heavens, and the mode of solution here alluded to could not be applied at all.

The equation here spoken of may no doubt be usefully applied in favourable circumstances, particularly when the comet

is at a considerable distance from the sun, and has a slow motion, allowing an interval of 20, or even a greater number of days, between the extreme observations; in which time the apparent deviation from a great circle of the heavens will be considerable enough to define the magnitude of ζ with tolerable certainty. But in many circumstances the same mode of solution cannot be applied with advantage or success: as when a comet has a quick motion, which will allow only a very short interval between the observations; or when a comet and the earth are both at the same distance from the sun, or nearly so, in which case ζ is evanescent, or very small.

It appears then, that, generally speaking, the problem of the comets must, in reality, be considered as indeterminate, if we set aside the condition derived from the nature of the parabola. In this respect there is an essential distinction between the investigation of the orbit of a planet and that of a comet; a distinction which, when due regard is had to practical utility, renders it necessary to separate the particular case from the general problem. When the question is to determine a planet's orbit by means of three observations made at short intervals of time, without any assistance from the supposition of a circular orbit, or other hypothesis, the equations which the data furnish are barely sufficient for finding three *radii vectores* of the orbit and the two angles contained between them; and there are no supernumerary conditions to compensate what may be imperfect in the observations. In whatever manner we proceed, the solution will lead to an equation, such as that we have been speaking of; and the success of the investigation will therefore depend upon our being able to ascertain with some degree of certainty the magnitude of the coefficients

which that equation contains. If there were the same uncertainty in this case that both theory and experience agree in shewing there is in the case of the comets, the problem, as applied to the planets, would be a mere theoretical speculation without any practical utility. It may therefore be asked, what are the precise circumstances which make the investigation successful in the one case, while in the other we must seek aid from the nature of the orbit in order to obtain a useful result? In the first place, the observations of the planets are susceptible of a much greater degree of accuracy than those of the comets; which confines the uncertainty arising from the errors of observation within much narrower limits in the one case than in the other. In the second place, the distance of a planet from the sun is never equal, or very nearly equal, to the earth's distance from the same body; and this occasions a greater curvature of the apparent path than often takes place with regard to the comets. We may likewise add that the motion of a planet, in an orbit of moderate eccentricity, is slower than the motion of a comet at a like distance from the sun; which allows a longer interval between the observations of a planet, and thereby contributes to heighten the effect arising from the inflection of the apparent path. It is to these causes, as well as to the excellence of his methods of investigation, that we must ascribe the great exactness with which the orbits of the new planets discovered in the course of the present century, have been determined by M. Gauss, in a work of great merit, *on the Theory of the Motion of the Celestial Bodies*, published in 1809, which cannot but add much to his reputation, already very high on account of former scientific discoveries.

7. If we put

$$\delta = \frac{1}{\tan.(c^\circ - n)} \cdot \left\{ \sin.(c^\circ - n) - \frac{\tan.\lambda^\circ}{\tan.i} \right\};$$

then the equation (9) will become by substitution

$$\sigma'' \cos.\lambda^\circ \cdot \delta \cdot \rho^\circ = \frac{\tau\tau'(\tau+\tau')}{2} \cdot \left(\frac{1}{r^{\prime 3}} - \frac{1}{R^{\prime 3}} \right) \cdot R^\circ \cos.(c^\circ - n):$$

and this value being substituted in the first of the equations (8), we shall get

$$\sigma'\rho \cos.h + \sigma\rho' \cos.h' = \sigma''\rho^\circ \cos.\lambda^\circ \cdot \left\{ \cos.(c^\circ - n) - \delta \right\}.$$

But, by putting $\cos.\lambda^\circ \tan.\lambda^\circ$ for $\sin.\lambda^\circ$, and then dividing by $\sin.i$, the third of the same set of equations will become

$$\sigma'\rho \sin.h + \sigma\rho' \sin.h' = \sigma''\rho^\circ \cdot \cos.\lambda^\circ \cdot \frac{\tan.\lambda^\circ}{\sin.i}.$$

Now if we make

$$\tan.\omega = \frac{1}{\sin.i} \times \frac{\tan.\lambda^\circ}{\cos.(c^\circ - n) - \delta};$$

then, by subtracting the two equations just found, after having multiplied them by $\sin.\omega$ and $\cos.\omega$ respectively; we shall get

$$\sigma'\rho \sin.(\omega - h) - \sigma\rho' \sin.(h' - \omega) = 0$$

whence

$$\rho' = \rho \times \frac{\sin.(\omega - b)}{\sin.(b - \omega)} \cdot \frac{\sigma'}{\sigma}.$$

The value of $\frac{\sigma'}{\sigma}$, obtained by substituting the series that σ' and σ stand for, will be as follows, viz.

$$\frac{\sigma'}{\sigma} = \frac{\tau'}{\tau} \cdot \left\{ 1 - \frac{\tau^2 - \tau'^2}{6r^{\circ 3}} \right\}.$$

In this formula $\frac{\tau^2 - \tau'^2}{6r^{\circ 3}}$ is evanescent when $\tau = \tau'$; and in all cases it is a very small quantity; because the intervals of time between the observations made use of for finding a comet's orbit, should in no instance be extremely unequal. If then we suppose $\frac{\sigma'}{\sigma} = \frac{\tau'}{\tau}$, we shall get these two formulas, viz.

$$\beta = \frac{\sin.(\omega - b)}{\sin.(b' - \omega)} \cdot \frac{\tau'}{\tau}$$
$$\rho' = \beta \cdot \rho; \qquad (11).$$

which are very exact, when the intervals between the observations are not too great.

The angle ω which enters into the preceding formulas, depends partly upon the value of the quantity δ; and it thus becomes impossible to assign the value of that angle when δ cannot be determined. If we compare δ with the quantity formerly denoted by ζ (No. 5), we shall get $\delta = \zeta \times \frac{\cos.(\epsilon^0 - \omega)}{\cos.\lambda^0}$: hence it appears that δ will be indeterminate when ζ is so; and in such cases therefore the last formulas will fail. Now the cases in which ζ becomes indeterminate have already been noticed (No. 5); they differ by a real distinction from the other cases of the problem, and require a separate discussion, if we wish to have clear and precise notions on the subject of this research: we shall therefore return to the examination of them in the sequel.

8. On account of the formulas (11), the coordinates of the comet at the two extreme observations, will depend only upon one unknown quantity, namely ρ: and if we substitute the values of the coordinates in the following expressions, viz. $r^2 = x^2 + y^2 + z^2$; $r'^2 = x'^2 + y'^2 + z'^2$; $V = xx' + yy' + zz'$; there will result these other expressions, which contain no unknown quantity but ρ, viz.

$$r^2 = R^2 + 2R \cos.\lambda \cos.(\epsilon - c) \cdot \rho + \rho^2$$
$$r'^2 = R'^2 + 2R'\beta \cos.\lambda' \cos.(\epsilon' - c') \cdot \rho + \beta^2 \rho^2$$
$$V = RR' \cos.(\epsilon' - \epsilon) + \left\{ R\beta \cos.\lambda' \cos.(\epsilon - c') + R' \cos.\lambda \times \cos.(\epsilon' - c) \right\} \times \rho + \beta \cos.\gamma \cdot \rho^2$$
$$\cos.\gamma = \cos.\lambda \cos.\lambda' \cos.(c' - c) + \sin.\lambda \sin.\lambda'.$$

We must now have recourse to the properties of the orbit to get such an equation between the functions, r^0, $r^{(2)}$ and V as shall serve to determine the unknown quantity which they contain. For this purpose we may employ the expression which gives the time of describing an arc of a parabola, by means of the chord of that arc and the two *radii vectores* drawn to its extremities. This elegant property of the motion in a parabola, seems to have been first found out by EULER, as M. GAUSS informs us; but it is commonly attributed to M. LAMBERT of Berlin, who probably discovered it without knowing what EULER had done, and who has extended it to all the conic sections.* It would be superfluous to give here the investigation of a truth so well known: it will be sufficient to refer to the *Mécanique Céleste* of LAPLACE,† or to the work of M. GAUSS. Let b denote the chord drawn between the places of the comet at the two extreme observations: then observing that $\tau + \tau'$ is the arc of the earth's mean motion corresponding to the time of describing the parabolic arc of which b is the chord; we shall get

$$\tau + \tau' = \tfrac{1}{6} \cdot \left\{ (r + r' + b)^{\frac{3}{2}} - (r + r' - b)^{\frac{3}{2}} \right\};$$

and, by expanding the radicals,

$$\tau + \tau' = \sqrt{r + r'} \cdot \left\{ \frac{b}{2} - \frac{1}{48} \cdot \frac{b^3}{(r+r')^2} - \frac{1}{256} \cdot \frac{b^5}{(r+r')^4} \text{ \&c.} \right\};$$

and, by squaring and omitting the sixth and higher powers of b,

$$4\,(\tau + \tau')^2 = (r + r') \cdot b^2 \cdot \left\{ 1 - \tfrac{1}{12} \cdot \frac{b^2}{(r+r')^2} \right\}.$$

Let $a^3 = 2r^0 + 2r^{(2)} = (r + r')^3 + (r - r')^3 =$

* Theor. Mot. Corp. Cœles. Lib. I. Sect. 3. § 106.

† Prem. Part. Lib. II. Chap. 4. § 27.

$(r+r')^2+\frac{(r^2-r'^2)^2}{(r+r')^2}$: then

$$r+r'=a\cdot\left\{1-\tfrac{1}{2}\cdot\left(\frac{r^2-r'^2}{a^2}\right)^2-\tfrac{1}{8}\cdot\left(\frac{r^2-r'^2}{a^2}\right)^4 \text{ \&c.}\right\}:$$

and by substituting this value in the foregoing expression we get, nearly,

$$4(\tau+\tau')^2=a\cdot b^3\cdot\left\{1-\tfrac{1}{12}\cdot\frac{b^3}{a^3}-\tfrac{1}{2}\cdot\left(\frac{r^2-r'^2}{a^2}\right)^2\right\};$$

consequently

$$b^3=\frac{4(\tau+\tau')^2}{a}\cdot\left\{1+\tfrac{1}{12}\cdot\frac{b}{a^3}+\tfrac{1}{2}\cdot\left(\frac{r^2-r'^2}{a^2}\right)^2\right\}:$$

which is exact to quantities of the sixth and higher orders.

Now $b^2=(x-x')^2+(y-y')^2+(z-z')^2=x^2+y^2+z^2-2(xx'+yy'+zz')+x'^2+y'^2+z'^2=r^2+r'^2-2V$: therefore in this last equation all the quantities concerned depend upon r^2, r'^2, and V, which themselves contain only one unknown quantity, namely ρ: thus that equation will serve to determine ρ, or the comet's distance from the earth at the time of the first observation, on the finding of which the solution of the problem depends.

9. If we now collect in one view all the formulas that have been investigated, we shall have the following method for computing the distance of a comet from the earth.

1st. The symbols τ and τ' denoting the intervals in days of mean time, between the middle observation and the first and second observations respectively; we begin with computing the arc of the earth's mean motion in the time $(\tau+\tau')$; this arc we shall now denote by θ, and it will be computed by this formula, viz. $\theta=\frac{2\varpi(\tau+\tau')}{365.25638}$; 2ϖ being the circumference of a circle whose radius is unit, and 365.25638 the length of the sydereal year; so that the logarithm of θ will be found by

adding the constant logarithm 8.2355814 to the logarithm of $r + r'$.

We must next compute the angles n and i, by the formulas investigated in No. 5: n being the longitude of one of the points in which a great circle drawn through the two extreme places of the comet cuts the ecliptic; namely, the longitude of that intersection of the two circles which immediately precedes the comet in longitude: and i being the inclination of the same great circle to the ecliptic.

We must further make these computations, viz.

$$\cos. h = \cos. \lambda \cos. (c - n)$$

$$\cos. h' = \cos. \lambda' \cos. (c' - n)$$

$$\delta = \frac{1}{\tan. (c^0 - n)} \cdot \left\{ \sin. (c^0 - n) - \frac{\tan. \lambda^0}{\tan. i} \right\}$$

$$\tan. \omega = \frac{1}{\sin. i} \times \frac{\tan. \lambda^0}{\cos. (c^0 - n) - \delta}$$

$$\beta = \frac{\sin. (\omega - b)}{\sin. (b' - \omega)} \cdot \frac{r'}{r}$$

$$\cos. \gamma = \cos. \lambda \cos. \lambda' \cos. (c' - c) + \sin. \lambda \sin. \lambda'.$$

2dly. We must reduce into numbers the following formulas, leaving ρ indeterminate, viz.

$$r^2 = R^2 + 2R \cos. \lambda \cos. (e - c) \cdot \rho + \rho^2$$

$$r'^2 = R'^2 + 2R'\beta \cos. \lambda' \cos. (e' - c') \cdot \rho + \beta^2 \cdot \rho^2$$

$$V = RR' \cos. (e' - e) + \left\{ R\beta \cos. \lambda' \cos. (e - c') + R' \cos. \lambda \cos. (e' - c) \right\} \cdot \rho + \beta \cos. \gamma \cdot \rho^2.$$

3dly. We must determine ρ by means of these formulas, viz.

$$b^2 = r^2 + r'^2 - 2V$$

$$a^2 = 2r^2 + 2r'^2$$

$$b' = \frac{4b^2}{a} \cdot \left\{ 1 + \frac{1}{12} \cdot \frac{b^2}{a^2} + \frac{1}{2} \cdot \left(\frac{r^2 - r'^2}{a^2} \right)^2 \right\}.$$

In the trials necessary for approximating to the value of ρ,

the terms of the final equation, which are of the fourth order, being always inconsiderable; they may be omitted at first, and then only taken into the account when a near value of ρ has already been obtained.

It has already been shown that $\delta = \zeta \cdot \frac{\cos.(\epsilon^\circ - n)}{\cos.\lambda^\circ}$, whence $\zeta = \frac{\delta}{\cos.(\epsilon^\circ - n)} \cdot \cos.\lambda^\circ$; therefore, since $\cos.\lambda^\circ$ is in every case positive, the quantity $\frac{\delta}{\cos.(\epsilon^\circ - n)}$ will have the same sign as ζ: and thus, from what is proved in No. 3, we infer that the comet's distance from the sun will be less or greater than the earth's distance, according as the sign of $\frac{\delta}{\cos.(\epsilon^\circ - n)}$ is positive or negative; and the two distances from the sun will be equal when $\frac{\delta}{\cos.(\epsilon^\circ - n)} = 0$. These observations will often enable us to assign such first values of ρ as will lead to a solution without many trials.

The preceding method will always give one solution. To prove this: let B denote the chord of the earth's orbit drawn between the places of the planet at the two extreme observations; and further, let

$$\cos. u = 1 - \frac{R + R' + B}{2}$$

$$\cos. u' = 1 - \frac{R + R' - B}{2}:$$

then if we apply to the earth's orbit the formula for finding the time of describing an arc of an ellipse by means of the chord of that arc, and the two radii vectores drawn to its extremities;* we shall get,

$$\theta = (u - \sin. u) - (u' - \sin. u').$$

Let $\cos. m = 1 - \frac{R + R'}{2}$: then $\cos. u = \cos. m - \frac{1}{2} B$; and

* Méc. Céleste, Prem. Part. Liv. II. § 27 formule *a*.

$\cos. u' = \cos. m + \frac{1}{2} B$: we may therefore suppose in series,

$u - \sin. u = M^{(0)} - M^{(1)} . \frac{B}{2} + M^{(2)} . \frac{B^2}{4} - M^{(3)} . \frac{B^3}{8} +$ &c.

$u' - \sin. u' = M^{(0)} + M^{(1)} . \frac{B}{2} + M^{(2)} . \frac{B^2}{4} + M^{(3)} . \frac{B^3}{8} +$ &c.

where $M^{(0)} = m - \sin. m$; and $M^{(1)}, M^{(2)}$ &c. are coefficients derived from $M^{(0)}$: hence,

$$\theta = M^{(1)} B + M^{3)} . \frac{B^3}{4} +$$ &c.:

and by squaring and neglecting the fourth and higher powers of B, we get,

$$B^2 = \frac{\theta^2}{M^{(1)}};$$

but, from the theory of series, we get

$$M^{(1)} = -\frac{d . (m - \sin. m)}{d . \cos. m} = \frac{1 - \cos. m}{\sin. m} = \sqrt{\frac{1 - \cos. m}{1 + \cos. m}};$$

therefore $\frac{1}{M^{(1)}} = \frac{1 + \cos. m}{1 - \cos. m} =$ (by substituting the value of $\cos. m$) $\frac{4}{R + R'} - 1$. Let $A^2 = 2R^2 + 2R'^2 = (R + R')^2 + (R - R')^2$; then we may substitute A for $R + R'$, since quantities of the fourth and higher orders are neglected in the value of B^2: consequently we shall have

$$B^2 = \frac{4\theta^2}{A} - \theta^2.$$

But, when quantities of the fourth order are neglected, the final equation of the foregoing method will become $b^2 = \frac{4\theta^2}{a}$: and B, R, R', A are what b, r, r', a become, when $\rho = o$; thus it appears that b^2 in less than $\frac{4\theta^2}{a}$, when $\rho = o$. But as ρ increases, at least after a certain limit, b^2 will increase without limit, and $\frac{4\theta^2}{a}$ will decrease without limit; so that, when

$\rho = \omega$, b^3 will be infinitely great, and $\frac{4\beta^3}{a}$ will be infinitely little. Therefore there will always be one value of ρ, between the limits o and ω, that will satisfy the equation $b^3 = \frac{4\beta^3}{a}$.

10. Before applying the method here proposed to examples, it will be necessary to investigate the formulas by which the elements of the orbit are to be computed.

Let v and v' denote the true anomalies of the comet at the two extreme observations; or the angles which r and r' make with the perihelion distance of the parabolic orbit; then in the triangle formed by r, r', and b, the angle opposite to b will be equal to the sum or difference of v and v': therefore $b^2 = r^2 + r'^2 - 2rr' \cos.(v \pm v')$: but it has already been shown that $b^2 = r^2 + r'^2 - 2V$: therefore $V = rr' \cos.(v \pm v')$. Let the angle contained by r and b in the same triangle, be denoted by φ; and that contained by r' and b, by φ': then it is plain that $\cos.\varphi = \frac{r - r' \cos.(v \pm v')}{b}$; and $\cos.\varphi' = \frac{r' - r \cos.(v \pm v')}{b}$: whence we get the following formulas for finding the angles φ and φ', which are the angles that r and r' make with the chord b, viz.

$$\cos.\varphi = \frac{r^2 - V}{r.b}$$

$$\cos.\varphi' = \frac{r'^2 - V}{r'.b};$$

observing that regard must be had to the signs of the cosines, as well as to their values in numbers. Again let

$$-\cos.\psi = \frac{r - r'}{b}$$

$$-\cos.\psi' = \frac{r' - r}{b};$$

then the angles ψ and ψ', the cosines of which differ only in their signs, will be supplements of one another; and, as it is

easy to prove from the nature of the parabola, they will be equal to the angles which the chord b makes with the axis of the orbit. From the angles thus found, the true anomalies are immediately deduced: for v is equal to the difference of ϕ and ψ; and v', to the difference of ϕ' and ψ'. Further, the time of passing the perihelion will be between the two extreme observations, when ψ and ψ' are both greater than φ and φ' respectively: otherwise the time of passing the perihelion will be before, or after, both the observations, according as r, the radius vector at the first observation is less, or greater, than r', the radius vector at the third observation. But these rules suppose that the angular motion of the comet in its orbit, in the interval between the extreme observations, is less than 180°; which, in fact, will comprehend all the cases that can occur in applying the method.

The true anomalies of r and r', being known, the perihelion distance, denoted by D, will be found by either of these formulas, viz.

$$D = r \cos.^2 \frac{v}{2}$$

$$D = r' \cos.^2 \frac{v'}{2}.$$

To find the time of passing the perihelion, we must take the times corresponding to v and v' from a table of the motion in a parabola; these times, being multiplied by $D^{\frac{3}{2}}$, will give the intervals between the two extreme observations and the time of passage.

In order to determine the position of the orbit in the heavens, it is best to begin with seeking the heliocentric latitudes: let these latitudes be l and l'; then

$$\text{sin. } l = \frac{t}{r} \text{ . sin. } \lambda; \quad \text{sin. } l' = \frac{\theta_t}{r'} \text{ . sin. } \lambda'.$$

Let E and E′ denote the two angles of elongation (or the differences of the geocentric longitudes of the sun and the comet); and C and C′, the two angles of commutation (or the differences in longitude of the earth and the comet): then

$$\text{sin. C} = \frac{r}{r'} \cdot \frac{\text{cos. } \lambda}{\text{cos. } l} \times \text{sin. E}$$

$$\text{sin. C}' = \frac{R}{r'} \cdot \frac{\text{cos. } \lambda'}{\text{cos. } l'} \times \text{sin. E}';$$

by means of which formulas, the heliocentric longitudes of the comet at the two extreme observations will be known.

Let ϒN*n* represent the ecliptic; K and K′, the places of the comet in the ecliptic, determined by the heliocentric longitudes already computed; PK and PK′, the circles of latitude, and KC and K′C′ the heliocentric latitudes already found; and lastly, let NCC′*n* be the great circle in which the plane of the orbit meets the heavens: then although it is possible that the comet may pass from C to C′, either by describing the small arc CC′, or the arc which CC′ wants of a whole circle; yet from the nature of the case, no ambiguity can hence arise, because in finding a first approximation to the orbit of a comet, the arc CC′, which embraces the whole motion in the orbit in the interval between the extreme observations, will never contain a great number of degrees: it is therefore easy to infer whether the comet has increased, or diminished its longitude, in passing from C to C′; in the first case, the motion of the comet is direct; in the second case, it is retrograde. Nor can there be any difficulty, or ambiguity, in distinguishing the *Ascending Node*, through which the comet passes from the south, to the north, side of the ecliptic. The ascending node is marked, in the figure, with N; the descending node, with *n*.

The arc CC′ is the measure of the angle at the sun's centre, contained by r and r'; and it is known by means of the true anomalies, since it is $= v \mp v'$; or, by means of the formula, cos. CC′ $= \frac{\mathrm{V}}{rr'}$: PC and PC′ are the complements of the heliocentric latitudes; and KK′ is the difference of the heliocentric longitudes. Now in the spherical triangle CPC′, we have

sin. CC′ : sin. P, or sin. KK′ : : sin. PC′, or cos. l' : sin. C;

and in the right-angled triangle KCN, of which the angle N is the inclination to the ecliptic, denoted by I; we have

Rad. : cos. KC, or cos. l : : sin. C : cos. I;

therefore by combining these two proportions, we get

$$\text{cos. I} = \frac{\text{sin. KK}'}{\text{sin. CC}'} \cdot \text{cos. } l \text{ cos. } l'.$$

The resolution of the right-angled spherical triangle KCN, will give the longitude of the ascending node, and the place of the perihelion on the orbit. For the side KN is the difference in longitude between the comet at the first observation and the ascending node: and the hypothenuse NC is the angular distance on the orbit between the same node and the comet at the first observation; and from this it is easy to find the angular distance between the node and the perihelion, which will fix the place of the perihelion on the orbit.

11. In applying the preceding method, the examples in LEGENDRE's Memoir and the supplement to it, have been purposely taken: first, because these make a selection of instances greatly varied in their circumstances: secondly, because the results obtained by the formulas investigated in this paper can thus be compared with his, and other methods, and with the corrected elements, which are likewise given in his Memoir.

Application to the second Comet of 1781.

The following observations of this comet, which was discovered by M. MECHAIN, are taken by LEGENDRE from the *Mém. de l'Acad. des Sciences*, for 1780:* they are all reduced to the same hour of every day, namely to $8^h\ 29'\ 44''$ mean time at Paris.

Times of Observation.	Longitude.	North Latitude.	Longitude of ⊕.	Log. of R.
September.				
14	$c,\ 307°\ 14'\ 45''$	$\lambda,\ 55°\ 17'\ 9''$	$e,\ 52°\ 54'\ 2''$	R, 9·994864
19	$c,''\ 306°\ 51'\ 26''$	$\lambda,''\ 39°\ 14'\ 48''$	$e,''\ 57°\ 57'\ 4''$	R,'' 9·994426
24	$c,'\ 306°\ 42'\ 20''$	$\lambda,'\ 31°\ 4'\ 52''$	$e,'\ 63°\ 0'\ 41''$	R,' 9·994028

By applying the directions and formulas of No. 8, I have found,

$$\text{Log. } 4b^2 = 9{\cdot}0732228$$
$$n = 306°\ 19'\ 5''$$
$$i = 89\ 21\ 27$$
$$\delta = -\ 0{\cdot}0000970$$
$$\omega = 39°\ 14'\ 49''$$
$$h = 55\ 17\ 28$$
$$h' = 31\ 5\ 0$$
$$\log.\ \beta = 0{\cdot}2892147$$
$$\cos.\ \gamma = 9{\cdot}9600258$$

With regard to the three principal formulas of the orbit, I remark that it is the logarithms of the coefficients, and not the coefficients themselves, that are afterwards used: on this

* Nouvelles Méthodes, p. 33, § xxxii.

account it is convenient to denote the coefficients by means of their logarithms: which is done, by writing, for instance, num. (9.482480) for the number whose logarithm is 9·482480. The formulas of this orbit, expressed in the manner here proposed, are as follows, viz.

$r^2 = 0{\cdot}976625 - \text{num.}\,(9{\cdot}482480) \times \rho + \rho^2$

$r'^2 = 0{\cdot}972873 - \text{num.}\,(0{\cdot}163532) \times \rho + \text{num.}\,(0{\cdot}578429) \times \rho^2$

$V = 0{\cdot}959610 - \text{num.}\,(9{\cdot}847341) \times \rho + \text{num.}\,(0{\cdot}249241) \times \rho^2$.

We are now prepared to approximate to the value of ρ; for which purpose we must seek a first value as near the truth as we can. Now, in this instance, $\frac{\delta}{\cos.\,(\phi^0 - n)}$ is positive, and very small: hence, we may infer that r will be less than R, but, approaching near to an equality with it. Let us then suppose $r^2 = R^2 = 0{\cdot}976625$; and we get $\rho = \text{num.}\,(9{\cdot}482480) = 0{\cdot}3087$; or $\rho = 0{\cdot}3$ nearly. By substituting $0{\cdot}3$ for ρ, we get

$r^2 = 0{\cdot}975508$

$r'^2 = 0{\cdot}876635$

$V = 0{\cdot}908288$

$b^2 = r^2 + r'^2 - 2V = 0{\cdot}035567$

$a^2 = 2r^2 + 2r'^2 = 3{\cdot}704286$ .. log. $0{\cdot}5687044$

log. $a = 0{\cdot}2843522$

$b^2 - \frac{4b^2}{a} = -0{\cdot}025932.$

The error here having the same sign as in the supposition $\rho = 0$ (No. 9); we hence learn that ρ is greater than $0{\cdot}3$. Suppose $\rho = 0{\cdot}4$, then

$r^2 = 1{\cdot}015136$

$r'^2 = 0{\cdot}996083$

$V = 0{\cdot}962188$

$b^{s} = 0{\cdot}086843$

$a^{s} = 4{\cdot}022438 \ldots \log. \ 0{\cdot}6044892$

$\log. \ a = 0{\cdot}3022446$

$b^{s} - \frac{4b^{s}}{a} = +\ 0{\cdot}027826.$

This error is nearly equal to the former, but it has an opposite sign: hence ρ will be nearly $= 0{\cdot}35$. Suppose $\rho = 0{\cdot}35$, then

$r^{s} = 0{\cdot}992821$

$r'^{s} = 0{\cdot}926888$

$V = 0{\cdot}930800$

$b^{s} = 0{\cdot}058109$

$a^{s} = 3{\cdot}839418 \ldots \log. \ 0{\cdot}5842654$

$\log. \ a = 0{\cdot}2921327$

$b^{s} - \frac{4b^{s}}{a} = -\ 0{\cdot}002298.$

By comparing this error with the last, we get a nearer value of ρ, viz. $\rho = 0{\cdot}354$: and the approximation is now far enough advanced to take in all the terms of the final equation in making the next substitution, viz. $\rho = 0{\cdot}354$: then

$r^{s} = 0{\cdot}994422$

$r'^{s} = 0{\cdot}931729$

$V = 0{\cdot}932985$

$b^{s} = 0{\cdot}060181$

$a^{s} = 3{\cdot}852302 \ldots \log. \ 0{\cdot}5857202$

$\log. \ a = 0{\cdot}2928601$

$1 + \frac{1}{2} \cdot \left(\frac{r^{s} - r'^{s}}{a^{s}}\right)^{2} + \frac{1}{12} \cdot \frac{b^{s}}{a^{s}} = 1{\cdot}001434$

$b^{s} - \frac{4b^{s}}{a} \times 1{\cdot}001434 = -\ 0{\cdot}000212.$

In order to get another equation to compare with this last, I resume the equation of the supposition, $\rho = 0{\cdot}350$, and correct it by taking in all the terms of the final equation: then

$$b^2 - \frac{4b^2}{a} \times 1{\cdot}001434 = -\,0{\cdot}002385:$$

and now by comparing the two errors I finally get $\rho = 0{\cdot}35439$. This value being substituted, the result is as follows, viz.

$$r^2 = 0{\cdot}994580$$
$$r'^2 = 0{\cdot}932205$$
$$V = 0{\cdot}933200$$
$$b^2 = 0{\cdot}060385$$
$$a^3 = 3{\cdot}853570 \ldots \log. \; 0{\cdot}5858633$$
$$\log. \; a = 0{\cdot}2929316$$
$$b^2 - \frac{4b^2}{a} \times 1{\cdot}001434 = +\,0{\cdot}000002.$$

Having now obtained the exact values of ρ, r^2, r'^2, V and b^2, it remains to deduce the elements of the orbit from them. Now we have (No. 10)

$$\frac{r^2 - V}{rb} = \cos. \phi \therefore \phi = 75^\circ\ 29'\ 48''$$
$$\frac{r - r'}{b} = -\cos. \psi \therefore \psi = 97\ 25\ 50$$
$$\text{Anomaly } v = 21\ 56\ 7$$
$$\frac{r'^2 - V}{r'b} = \cos. \phi' \therefore \phi' = 90^\circ\ 14'\ 25''$$
$$\frac{r - r'}{b} = -\cos. \psi' \therefore \psi' = 82\ 34\ 10$$
$$\text{Anomaly } v' = 7\ 40\ 15.$$

For the perihelion distance, we have

$$D = r \cos.^2 \tfrac{v}{2} \therefore \log. D = 9{\cdot}9828082$$
$$D = r' \cos.^2 \tfrac{v'}{2} \therefore \log. D = 9{\cdot}9828083$$
$$D = 0{\cdot}961188.$$

By a table of the motion in a parabola I have found the intervals corresponding to v and v', as follows, viz.,

	16·1316 days,	and 5·5198 days
log.	1·2076773	0·7419883
log. $D^{\frac{1}{2}}$ =	9·9742193	9·9742193
	1·1818896	0·7161356
	Num. 15·2016	Num. 5·2016.

These are the intervals between the two extreme observations and the passage of the perihelion, which according to the rule in No. 10, will be posterior to both the observations: therefore the comet will be in the perihelion, Nov. 29·5556, or

Nov. 29, 13^h 20′ 4″.

M. MECHAIN has applied the method of LAPLACE to this comet, using as the basis of his calculations, five observations between the 24th and 25th of November.* LAPLACE's method gives rules for finding two elements only, viz. the perihelion distance, and the time of arriving at the perihelion: for the sake of comparison, I now subjoin the results of M. MECHAIN; the same elements calculated by LEGENDRE from the observations used here; and the more exact elements corrected by distant observations.

	Perihelion Distance.	Time of Passage Nov. 29.
		h ′ ″
M. MECHAIN by LAPLACE's method	0·9588509	18 10 84
LEGENDRE	0·960449	16 18 19
In this paper	0·961188	13 20 4
Corrected elements	0·960995	12 42 46

* These calculations are given in BIOT's Astron. 2d Edit. Vol. III.

I have likewise computed the remaining elements of this orbit by the rules laid down in No. 10, and I have found them to be as follows, viz.

Inclination - -	27° 1′ 00″
Place of the ascending node	77 56 32
Place of the perihelion -	15 59 18

The method here proposed will represent the two extreme observations exactly; so that all the error of the solution will fall upon the middle observation. I have therefore employed the elements found above, to compute the geocentric place of the comet, Nov. 19, 8h 29′ 44″, in order to compare it with observation. The result of this calculation is as follows, viz.

Calculated Longitude.	Observed Longitude.	Error.	Calculated Latitude.	Observed Latitude.	Error.
° ′ ″	° ′ ″	′	° ′ ″	° ′ ″	′
306 51 30	306 51 26	+ 4	89 14 34	89 14 48	— 14

In LEGENDRE's memoir I find a like comparison of the corrected elements with observation; the error in longitude being + 100″; and the error in latitude, — 55″. It appears then that the errors of the elements found here are almost nothing, and even much less than those of the corrected elements.

Application to the Comet of 1769.

The places of this comet set down below are not derived from actual observation; they are calculated by LEGENDRE from the known elements of the orbit.* In this instance therefore, the inaccuracy of the results is to be attributed, not to

* Nouvelles Méthodes, p. 43, § XL.

the errors of observation, but solely to the defects of the method.

Times Sept.	Longitudes.	South Latitudes	Longitudes of ⊕.	Log. R.
d. h.	☌, ° ′ ″	λ, ° ′ ″	☉, ° ′ ″	
8 14	☌, 101 18 8	λ, 22 14 35	☉, 346 35 31	R, 0·0026648
10 14	☌°, 112 51 31	λ°, 23 28 15	☉°, 348 32 22	
12 14	☌′, 124 26 47	λ′, 23 48 36	☉′, 350 29 20	R′, 0·0021838

From these data, I have found

$$\log. 4\theta^2 = 8{\cdot}2773428$$
$$n = 33^\circ\ 22'\ 56''$$
$$i = 23\ 48\ 49$$
$$\delta = +\ 0{\cdot}0006684$$
$$\omega = 80^\circ\ 23'\ 45''$$
$$h = 69\ 38\ 21\tfrac{1}{2}$$
$$h' = 90\ 58\ 25$$
$$\log. \beta = 0{\cdot}0071924$$
$$\cos. \gamma = 9{\cdot}9691708.$$

$r^2 = 1{\cdot}012347 -$ num. $(9{\cdot}8913194) \times \rho + \rho^2$

$r'^2 = 1{\cdot}010107 -$ num. $(0{\cdot}1132123) \times \rho +$ num. $(0{\cdot}0143848) \times \rho^2$

$V = 1{\cdot}008888 -$ num. $(0{\cdot}0104917) \times \rho +$ num. $(9{\cdot}9763632) \times \rho^2$.

In this instance $\frac{\delta}{\cos.(\varepsilon''-\varepsilon)}$ is positive: therefore the comet is nearer the sun than the earth: hence $\rho <$ num. $(9{\cdot}8913194)$, or ρ $0{\cdot}778$. In order to get narrower limits, I take $\rho = \frac{0{\cdot}778}{2} = 0{\cdot}389$, or $\rho = 0{\cdot}4$, for a first approximation; then

$r^2 = 0.860903$

$r'^2 = 0.656370$

$V = 0.750632$

$b^2 = r^2 + r'^2 - 2V = 0.016009$

$a^2 = 2r^2 + 2r'^2 = 3.034546 \ldots$ log. 0.4820936

log. $a = 0.2410468$

$b^2 - \frac{4b^2}{a} = +0.005137.$

As this error is positive ρ must be diminished: therefore suppose $\rho = 0.3$; then

$r^2 = 0.868764$

$r'^2 = 0.713794$

$V = 0.786784$

$b^2 = 0.008990$

$a^2 = 3.165116 \ldots$ log. 0.5003896

log. $a = 0.2501948$

$b^2 - \frac{4b^2}{a} = -0.001655.$

By comparing the two errors it will appear that ρ is between 0.32 and 0.33: suppose $\rho = 0.33$, and because the errors are now small, let all the terms of the final equation be taken into account; then

$r^2 = 0.864306$

$r'^2 = 0.694896$

$V = 0.773950$

$b^2 = 0.010809$

$a^2 = 3.117404 \ldots$ log. 0.4937980

log. $a = 0.2468965$

$1 + \frac{1}{3}\left(\frac{r^2 - r'^2}{a^2}\right)^2 + \frac{1}{12} \cdot \frac{b^2}{a^2} = 1.001774$

$b^{\mathrm{IV}} - \frac{4b^{\mathrm{II}}}{a} \times 1{\cdot}001774 = + 0{\cdot}000057.$

Because the last value of ρ is too great, let $\rho = 0{\cdot}329$: then

$r^{\mathrm{II}} = 0{\cdot}864426$

$r'^{\mathrm{II}} = 0{\cdot}695012$

$\mathrm{V} = 0{\cdot}774351$

$b^{\mathrm{II}} = 0{\cdot}010736$

$a^{\mathrm{II}} = 3{\cdot}118876 \ldots \log. 0{\cdot}4939875$

$\log. a = 0{\cdot}2469938$

$b^{\mathrm{IV}} - \frac{4b^{\mathrm{II}}}{a} \times 1{\cdot}001774 = - 0{\cdot}000007.$

By comparing the two last errors we finally get $\rho = 0{\cdot}32911$: then

$r^{\mathrm{II}} = 0{\cdot}864412$

$r'^{\mathrm{II}} = 0{\cdot}694945$

$\mathrm{V} = 0{\cdot}774307$

$b^{\mathrm{II}} = 0{\cdot}010748$

$a^{\mathrm{II}} = 3{\cdot}118714 \ldots \log. 0{\cdot}4939760$

$\log. a = 0{\cdot}2469880$

$b^{\mathrm{IV}} - \frac{4b^{\mathrm{II}}}{a} \times 1{\cdot}001774 = 0.$

From the values just found, we get

$\log. r = 9{\cdot}9683604$

$\log. r' = 9{\cdot}9209752$:

and the angle contained by r and r', the cosine of which is $= \frac{\mathrm{V}}{rr'}$, will thence be found $= 2^{\circ}\ 31'\ 35''$. These quantities are sufficient for determining the parabola described by the comet; and as they are the immediate results of the method, the fairest way of judging of the exactness of that method, seems to be, to compare them with the true values calculated from the

known elements of the orbit. Now the elements employed by LEGENDRE in computing the places of the comet used in this example, are these, viz.

log. of perihelion dist. 9·090847
Time of passage, Oct. 7·5810;

from which data I have calculated the radii vectores and the true anomalies for the 8th and 12th of September at 14^h, as follows, viz.

September.	Log. of Rad. Vect.	Anomaly.
d. h. 8 14	9·968361	° ′ ″ 137 17 35
12 14	9·920961	134 46 10
Angle between r and r'		2 31 25

These quantities differ very little from the results obtained above: the errors are indeed hardly greater than the discrepancies which it is difficult to avoid in a complicated calculation, viz. first computing the geocentric longitudes and latitudes, and then going back from these to the elements of the orbit.

The exactness of the approximation is sufficiently proved by the comparison already made: but I have likewise calculated these two elements, viz.

log. of perihelion dist. - 9·091920
Time of passage, Oct. - 7·5403

the perihelion distance being $\frac{1}{400}$th part too much, and the time of passage about $13\frac{1}{2}'$ later than the true time. These errors, although very small, are yet greater than might be

expected, considering the exactness with which the comet's distances from the sun, and the included angle, were determined: but a very little variation of these quantities will produce a great alteration in the magnitude of the parabola, when the angle contained by the *radii vectores* is so small as it is in this instance.

Of the two comets to which the method in this Paper has been applied, the first varies very slowly in longitude, and very rapidly in latitude; the second, on the contrary, has a quick motion in longitude, and a very slow motion in latitude: in both instances however the intervals between the observations are equal; and I now proceed to give other examples where this condition is not observed.

Application to the second Comet of 1805.

The following observations are the same that LEGENDRE has used for determining the orbit of this comet:* they were communicated to him by M. BOUVARD.

Times November	Longitudes.	North Latitudes.	Longitude of ☉.	Log. R.
23·322411	c, 24° 41′ 5″	λ, 27° 25′ 19″	e, 61° 8′ 23″	R, 9·9942042
30·51095	c'', 15 38 36	λ'', 19 25 6	e'', 68 25 47	R″, 9·9986655
December 5·29581	c', 2 6 38	λ', 3 19 22	e', 73 17 5	R′, 9·9933784

Calculating from these data, we get

$$\log. 4\theta^{\circ} = 9{\cdot}2296576$$
$$n = 359^{\circ}\,21'\,58\tfrac{1}{2}''$$

* Supplément aux nouvelles Méthodes, § 30, p. 36.

$$i = 50^\circ\ 30'\ 11''$$
$$\delta = -\ 0{\cdot}0039918$$
$$\omega = 25^\circ\ 21'\ 33''$$
$$h = 36\ \ 38\ \ 32\tfrac{1}{2}$$
$$h' = \ \ 4\ \ 18\ \ 27$$
$$\log.\ \tau = 0{\cdot}8566407$$
$$\log.\ \tau' = 0{\cdot}6798693$$
$$\log.\ \beta = 9{\cdot}5593803$$
$$\cos.\ \gamma = 9{\cdot}9268248$$

$r^2 = 0{\cdot}973668 + \text{num.}\,(0{\cdot}1489016) \times \rho + \rho^2$

$r'^2 = 0{\cdot}969967 + \text{num.}\,(9{\cdot}3618159) \times \rho + \text{num.}\,(9{\cdot}1187606) \times \rho^2$

$V = 0{\cdot}950068 + \text{num.}\,(9{\cdot}8819026) \times \rho + \text{num.}\,(9{\cdot}4862051) \times \rho^2$.

In this instance, the form alone of r^2 shows that $r > R$; and likewise that r increases continually as ρ increases: we will therefore make different substitutions for ρ, increasing the value when the error is negative, and decreasing it when the error is positive.

First, let $\rho = 0{\cdot}1$: then

$$r^2 = 1{\cdot}124556$$
$$r'^2 = 0{\cdot}994286$$
$$V = 1{\cdot}029816$$
$$b^2 = r^2 + r'^2 - 2V = 0{\cdot}060210$$
$$a^2 = 2r^2 + 2r'^2 = 4{\cdot}237684 \ldots \log.\ 0{\cdot}6271286$$
$$\log.\ a = 0{\cdot}3135643$$
$$b^2 - \frac{4b^2}{a} = -\ 0{\cdot}022221.$$

Next, let $\rho = 0{\cdot}2$; then

$$r^2 = 1{\cdot}295456$$
$$r'^2 = 1{\cdot}021234$$
$$V = 1{\cdot}114698$$
$$b^2 = 0{\cdot}087294$$

$$a^2 = 4{\cdot}633380 \ldots \log. \; 0{\cdot}6658978$$

$$\log. \; a = 0{\cdot}3329489$$

$$b^2 - \tfrac{4b^3}{a} = + \; 0{\cdot}008461.$$

A comparison of the two errors will show that $\rho = 0{\cdot}17$ nearly: this value being substituted, we get

$$r^2 = 1{\cdot}242087$$

$$r'^2 = 1{\cdot}012874$$

$$V = 1{\cdot}088439$$

$$b^2 = 0{\cdot}078083$$

$$a^2 = 4{\cdot}509922 \ldots \log. \; 0{\cdot}6541690$$

$$\log. \; a = 0{\cdot}3270845$$

$$b^2 - \tfrac{4b^3}{a} = - \; 0{\cdot}001822.$$

By comparing this error with the last, it will be found that ρ is between $0{\cdot}175$ and $0{\cdot}176$: I therefore put $\rho = 0{\cdot}176$; and as the errors are now small, I employ the complete final equation: then

$$r^2 = 1{\cdot}252617$$

$$r'^2 = 1{\cdot}014527$$

$$V = 1{\cdot}098647$$

$$b^2 = 0{\cdot}079850$$

$$a^2 = 4{\cdot}534288 \ldots \log. \; 0{\cdot}6565091$$

$$\log. \; a = 0{\cdot}3282546$$

$$1 + \tfrac{1}{2} \cdot \left(\tfrac{r^2 - r'^2}{a^2}\right)^2 + \tfrac{1}{12} \cdot \tfrac{b^2}{a^2} = 1{\cdot}002846$$

$$b^2 - \tfrac{4b^3}{a} \times 1{\cdot}002846 = - \; 0{\cdot}000067.$$

I next put $\rho = 0{\cdot}177$: then

$$r^2 = 1{\cdot}254378$$

$$r'^2 = 1{\cdot}014803$$

$$V = 1{\cdot}094516$$
$$b^2 = 0{\cdot}080149$$
$$a^2 = 4{\cdot}538362 \ldots \text{log. } 0{\cdot}6568991$$
$$\text{log. } a = 0{\cdot}3284496$$
$$b^2 - \tfrac{4b^2}{a} \times 1{\cdot}002846 = +\,0{\cdot}000268.$$

From the two last errors we finally get $\rho = 0{\cdot}17620$; which must be substituted: then

$$r^2 = 1{\cdot}252969$$
$$r'^2 = 1{\cdot}014582$$
$$V = 1{\cdot}093821$$
$$b^2 = 0{\cdot}079909$$
$$a^2 = 4{\cdot}535102 \ldots \text{log. } 0{\cdot}6565888$$
$$\text{log. } a = 0{\cdot}3282944$$
$$b^2 - \tfrac{4b^2}{a} \times 1{\cdot}002846 = o.$$

From these values we get

$$\tfrac{r^2 - V}{rb} = \cos.\ \varphi \therefore \varphi = \ 59^\circ\ 48'\ 14''$$
$$\tfrac{r'^2 - V}{r'b} = \cos.\ \varphi' \therefore \varphi' = 106\ \ 9\ 30$$
$$\tfrac{r - r'}{b} = -\cos.\ \psi = +\cos.\ \psi' \therefore \begin{cases} \psi = 113^\circ\ 21'\ 45'' \\ \psi' = \ 66\ 38\ 15 \end{cases}$$
$$\text{anomalies} \begin{cases} v = \psi - \varphi = 53^\circ\ 33'\ 31'' \\ v' = \varphi' - \psi' = 39\ 31\ 15 \end{cases}$$

Then, to find the perihelion distance D, we have

$$D = r \cos.^2 \tfrac{v}{2} = 9{\cdot}9504285 \text{ logarithms}$$
$$D = r' \cos.^2 \tfrac{v'}{2} = 9{\cdot}9504285.$$

The times corresponding to the anomalies, taken from a table of the comets, are as follows, viz.

days	45·0135	days	30·8044
log.	1·6533428	log.	1·4886127
log. D½	9·9256427	log. D½	9·9256427
	1·5789855		1·4142554
Num.	37·9303	Num.	25·9570

hence the time of passing the perihelion is fixed for Dec. 31·2527.

By making the rest of the computations in No. 10, I have found the following elements of this comet, viz.

Log. perih. dist. -	9·9504129
Time of passage,	Dec. 25·2527
Inclination -	16° 30′ 37½″
Place of ascend. node	250 33 34
Place of perihelion	109 20 21
Motion	Direct.

LEGENDRE, from a comparison of all the observations of this comet, which are only six in number, has found the following corrected elements, viz.

Log. perih. dist. -	9·9502700
Time of passage,	Dec. 25·28551
Inclination -	16° 31′ 10″
Place of ascend. node	250 33 34
Place of perihelion	109 23 39

which differ very little from those found above, and prove the exactness of the approximation by the method in this paper.

Neither of these two systems of elements represent the observation of Nov. 30, with much accuracy, the errors in longitude amounting to between + 2′½ and + 3′: on which account it is probable that there is some peculiar inexactness in that observation.

Application to the first Comet of 1805.

This comet was discovered by M. BOUVARD: and LEGENDRE has given us five observations communicated to him by that astronomer.* Of these observations those which I have selected for the purpose of computing the orbit, are unfavourable in several respects. For, besides that the intervals between them are too unequal, the whole time elapsed, which embraces a heliocentric motion of no less than 25°, is rather too long: and it happens that the comet is placed with regard to the earth and the sun, so as to approach very near the limit, when, as has been remarked in No. 9, the method will fail. A better choice cannot however be made from the observations recorded by LEGENDRE, without interpolating.

Times.	Longitudes.	North Latitudes.	Longitudes of ⊕.	Log. R.
October. 22·6849	c, 163° 20′ 58″	λ, 22° 59′ 53″	e, 29° 19′ 50″	R, 9·9974921
30·6867	c'', 183 48 32	λ'', 15 37 21	e'', 37 19 48	R″, 9·9964496
November. 3·7209	c', 191 46 15	λ', 12 2 29	e', 41 22 26	R′, 9·9960053

In this instance, we get

$$\log. 4\beta^2 = 9.2341873$$
$$n = 34^\circ\ 58'\ 31\tfrac{1}{2}''$$
$$i = 28\ 25\ 49$$
$$\delta = +\ 0.0246945$$
$$\omega = 146^\circ\ 17'\ 25''$$
$$h = 124\ 51\ 1$$
$$h' = 154\ 0\ 41$$

* Supplément, p. 14, § 15.

$$\log. r = 0.9031887$$
$$\log. r' = 0.6057574$$
$$\log. \beta = 0.1372458$$
$$\cos. \gamma = 9.9411400$$

$r^3 = 0.988192 - \text{num.}(0.104392) \cdot \rho + \rho^3$

$r'^3 = 0.981987 - \text{num.}(0.363921) \cdot \rho + \text{num.}(0.274492) \cdot \rho^3$

$V = 0.963404 - \text{num.}(0.244138) \cdot \rho + \text{num.}(0.078386) \cdot \rho^3.$

In this instance $\frac{\beta}{\cos.(\epsilon^0 - \pi)}$ is positive; and therefore the comet is nearer the sun than the earth is: hence $\rho <$ num. (0.104392), or $\rho < 1.270$. To get narrower limits I assume $\rho = \frac{1.27}{2} = 0.635$, or $\rho = 0.6$; then

$$r^3 = 0.585160$$
$$r'^3 = 0.272317$$
$$V = 0.341949$$
$$b^3 = 0.173579$$
$$a^3 = 1.714954 \ldots \log. 0.2342388$$
$$\log. a = 0.1171194$$
$$b^3 - \frac{4b^3}{a} = +0.042642.$$

This error being positive, the value of ρ must be diminished: therefore suppose $\rho = 0.5$; then

$$r^3 = 0.602332$$
$$r'^3 = 0.296528$$
$$V = 0.385635$$
$$b^3 = 0.127590$$
$$a^3 = 1.797720 \ldots \log. 0.2547220$$
$$\log. a = 0.1273610$$
$$b^3 - \frac{4b^3}{a} = -0.000297.$$

Here the error being very small, it will be proper to take in all the terms of the final equation: then,

$$1 + \tfrac{1}{2} \cdot \left(\frac{r^2 - r'^2}{a}\right)^2 \times \tfrac{1}{12} \cdot \frac{b^2}{a^2} = 1{\cdot}020382$$

$$b^2 - \frac{4b^2}{a} \times 1{\cdot}020382 = -\,0{\cdot}002903.$$

It now appears that ρ is between 0·6 and 0·5, but much nearer to 0·5: therefore put $\rho = 0{\cdot}51$; then

$$r^2 = 0{\cdot}599714$$
$$r'^2 = 0{\cdot}292413$$
$$V = 0{\cdot}380190$$
$$b^2 = 0{\cdot}131747$$
$$a^2 = 1{\cdot}784254 \ldots \log.\ 0{\cdot}2514566$$
$$\log.\ a = 0{\cdot}1257283$$

$$1 + \tfrac{1}{2} \cdot \left(\frac{r^2 - r'^2}{a}\right) + \tfrac{1}{12} \cdot \frac{b^2}{a^2} = 1{\cdot}020984$$

$$b^2 - \frac{4b^2}{a} \times 1{\cdot}20984 = +\,0{\cdot}000685.$$

By comparing this error with the last we get $\rho = 0{\cdot}5081$, which must be substituted; then

$$r^2 = 0{\cdot}600197$$
$$r'^2 = 0{\cdot}293163$$
$$V = 0{\cdot}381205$$
$$b^2 = 0{\cdot}130950$$
$$a^2 = 1{\cdot}786720 \ldots \log.\ 0{\cdot}2520565$$
$$\log.\ a = 0{\cdot}1260283$$

$$b^2 = \frac{4b^2}{a} \times 1{\cdot}02086 = -\,0{\cdot}000005.$$

The approximation has already been pushed farther than was requisite: for, in this instance, the terms of the fourth order affect the errors in the third place of decimal figures;

and therefore the terms of the higher orders, which are omitted, may be supposed to affect the errors in the fourth and fifth places of figures, beyond which degree of exactness the approximation is already carried. To compute the elements of the orbit, we have

$$\frac{r^2-V}{rb} = \cos.\varphi \therefore \varphi = \quad 38^\circ\ 38'\ \ 6''$$

$$\frac{r-r'}{r} = -\cos.\psi \therefore \psi = 130\ \ 8\ 21\tfrac{1}{2}$$

$$\text{Anomaly } v = \psi - \varphi = 91\ 30\ 15\tfrac{1}{2}$$

$$\frac{r'^2-V}{r'b} = \cos.\varphi' \therefore \varphi' = 116^\circ\ 42'\ \ 6\tfrac{1}{2}''$$

$$\frac{r'-r}{b} = -\cos.\psi' \therefore \psi' = \quad 49\ 51\ 38\tfrac{1}{2}$$

$$\text{Anomaly } v' = \varphi' - \psi' = 66\ 50\ 28.$$

Then, $D = r\cos.^2\frac{v}{2} = 9.57656635$ logarithms

$$D = r'\cos.^2\frac{v'}{2} = 9.57656635.$$

The times corresponding to the anomalies, taken from the table, are

days	114·048	days	62·1248
log.	2·0570877	log.	1·7932650
log. $D^{\frac{1}{2}}$,	9·3648453	log. $D^{\frac{1}{2}}$,	9·3648453
	1·4219330		1·1581103
Num.	26·4200	Num.	14·3916:

the difference of these two intervals is 12·0284; but it should be 12·0360: I therefore add 0·0038 to the first, and subtract it from the second interval; then they are 26·4238 and 14·3878: and by adding these corrected intervals, to the times of the observations, the time of the comet's passing the perihelion will come out, Nov. 18·1087.

Having completed the rest of the calculations in No. 10, the elements of this comet will be found as follows: viz.

Log. of perih. dist. -	9·5765685
Time of passage, Nov. -	18·1087
Inclination - -	15° 40′ 17″
Place of ascend. node -	345 7 39
Place of peri. -	148 11 7
	Motion direct.

These elements will not represent the two extreme observations with perfect accuracy as in the other examples, on account of the correction which it was necessary to introduce, in order to annihilate the difference between the time in the orbit and the actual interval between the observations: but the errors proceeding from this cause will be inconsiderable, and can hardly amount to 20″. If we employ the same elements to calculate the place of the comet, Oct. 30·6867, we shall find these results, viz.

	Geo. Long.	Geo. Lat.
Calculated	183° 51′ 20″	15° 36′ 37″
Observed	183 48 32	15 37 21
	error + 2 48	error − 44

These errors are in reality less than might be expected, considering the disadvantages under which the computation of this orbit is made. LEGENDRE has given a system of elements which represents the observations of the comet very exactly, the greatest error not exceeding 1′: and, for the sake of comparison, I likewise subjoin these elements, viz.

Log. of peri. dist. -	9·574798
Time of passage, Nov. -	18·01736
Inclination - -	15° 58′ 12″

Place of ascend. node	-	345°	6′ 51″
Place of the peri.	-	149	0 28

12. The method of finding a first approximation to a comet's orbit, investigated in this Paper, has now been applied to four examples in very different circumstances from one another; and the results obtained in them all are satisfactory. These instances will suffice for making the rules of calculation clear, and for confirming the accuracy of the analysis from which they have been deduced. It remains that we now consider the particular cases, already noticed (No. 7), in which this method fails.

If a new planet should be discovered that moved exactly in the plane of the ecliptic, three geocentric observations of it would not be sufficient for finding its orbit. The latitudes being wanting in the circumstances here supposed, the longitudes alone would not furnish conditions enow for determining the magnitude and position of the curve which the planet described. The same thing will likewise happen in another situation, when the latitudes, although not evanescent, yet depend, all of them in the same way, upon the longitudes; and that is when the three geocentric places of the planet are situated in one great circle of the heavens which cuts the ecliptic in the points occupied by the earth and the sun at the time of the middle observation. This last case may indeed be considered as including the former one, when the plane of the planet's orbit coincides with the ecliptic.

With regard to the comets it must be remembered that three complete observations are more than sufficient for finding an orbit: and on this account there are still conditions enow, but no more than enow, to determine the elements sought, in the

circumstances abovementioned in which the problem becomes indeterminate when applied to the planets. We shall therefore proceed to analyze this case, which has already been noticed as forming a real distinction in the problem we are considering.

Resume the three following equations which have already been investigated in No. 4, viz.

$$\sigma'\rho \cos. h + \sigma\rho' \cos. h' - \sigma''\rho^{\circ} \cos. \lambda^{\circ} \cos. (c^{\circ} - n) =$$
$$- \tfrac{\tau\tau'(\tau+\tau')}{2} \cdot \left(\tfrac{1}{r^{03}} - \tfrac{1}{R^{03}}\right) \cdot R^{\circ} \cos. (c^{\circ} - n)$$

$$\sigma'\rho \cos. i \sin. h + \sigma\rho' \cos. i \sin. h' - \sigma''\rho'' \cos. \lambda^{\circ} \sin. (c^{\circ} - n) =$$
$$- \tfrac{\tau\tau'(\tau+\tau')}{2} \cdot \left(\tfrac{1}{r^{03}} - \tfrac{1}{R^{03}}\right) \cdot R^{\circ} \sin. (c^{\circ} - n)$$

$$\sigma'\rho \sin. i \sin. h + \sigma\rho' \sin. i \sin. h' - \sigma''\rho^{\circ} \sin. \lambda^{\circ} = 0:$$

and employing h° and i° to denote the same things as in No. 5, let the following values found there, viz.

$$\cos. \lambda^{\circ} \sin. (c^{\circ} - n) = \cos. i^{\circ} \sin. h^{\circ}$$
$$\sin. \lambda^{\circ} = \sin. i^{\circ} \sin. h^{\circ}$$

be substituted in the two last of the equations above; then these equations will become,

$$\sigma'\rho \cos. i \sin. h + \sigma\rho' \cos. i \sin. h - \sigma''\rho^{\circ} \cos. i^{\circ} \sin. h^{\circ} =$$
$$- \tfrac{\tau\tau'(\tau+\tau')}{2} \cdot \left(\tfrac{1}{r^{03}} - \tfrac{1}{R^{03}}\right) \cdot R^{\circ} \sin. (c^{\circ} - n)$$

$$\sigma'\rho \sin. i \sin. h + \sigma\rho' \sin. i \sin. h - \sigma''\rho^{\circ} \sin. i^{\circ} \sin. h^{\circ} = 0:$$

further let these two equations be added together, after being multiplied by cos. i and sin. i respectively: then we shall get

$$\sigma'\rho \sin. h + \sigma\rho' \sin. h' - \sigma''\rho^{\circ} \cos. (i - i^{\circ}) \sin. h^{\circ} =$$
$$- \tfrac{\tau\tau'(\tau+\tau')}{2} \cdot \left(\tfrac{1}{r^{03}} - \tfrac{1}{R^{03}}\right) R^{\circ} \cos. i \sin. (c^{\circ} - n).$$

In the particular cases we are considering, either $i = i^{\circ}$; or i and i° are both evanescent; in both which cases cos. $(i - i^{\circ}) = 1$. And if we attend to the analysis in No. 5, it will appear

that, in every case, sin. $(i - i^\circ)$ is of the second order; and hence cos. $(i - i^\circ)$, differing from 1 only by quantities of the fourth order, may generally be regarded as equal to unit. The last equation will therefore become,

$$\sigma'\rho \sin. h + \sigma\rho' \sin. h' - \sigma''\rho^\circ \sin. h^\circ =$$
$$-\frac{\tau\tau'(\tau+\tau')}{2} \cdot \left(\frac{1}{r^3} - \frac{1}{R^3}\right) \cdot R^\circ \cos. i \sin. (\epsilon^\circ - n):$$

and, if this equation be combined with the first of the three equations set down at the beginning of this article,* so as to exterminate ρ° and ρ', we shall get,

$$\sigma'\rho \sin. (h^\circ - h) - \sigma\rho' \sin. (h' - h^\circ) = -\frac{\tau\tau'(\tau+\tau')}{2} \cdot \left(\frac{1}{r^3} - \frac{1}{R^3}\right) \cdot R^\circ \times$$
$$\left\{\cos. (\epsilon^\circ - n) \sin. h^\circ - \cos. i \sin. (\epsilon^\circ - n) \cos. h^\circ\right\};$$
$$\sigma'\rho \sin. (h' - h) - \sigma''\rho^\circ \sin. (h' - h^\circ) = -\frac{\tau\tau'(\tau+\tau')}{2} \cdot \left(\frac{1}{r^3} - \frac{1}{R^3}\right) \cdot R^\circ \times$$
$$\left\{\cos. (\epsilon^\circ - n) \sin. h' - \cos. i \sin. (\epsilon^\circ - n) \cos. h'\right\}.$$

Observing then that $\frac{\sigma'}{\sigma} = \frac{\tau'}{\tau}$, and $\frac{\sigma'}{\sigma''} = \frac{\tau'}{\tau+\tau'}$, nearly; if we put

$$B = \frac{\tau'(\tau+\tau')}{2} \cdot R^\circ \cdot \frac{\cos. (\epsilon^\circ - n) \sin. b^\circ - \cos. i \sin. (\epsilon^\circ - n) \cos. b^\circ}{\sin. b' - b^\circ};$$
$$\varepsilon = \frac{\sin. (b' - b)}{\sin. (b' - b^\circ)} \cdot \frac{\tau'}{\tau+\tau'};$$

we shall get,

$$\rho^\circ = \varepsilon \cdot \rho$$
$$\rho' = \rho \times \frac{\sin. (b^\circ - b)}{\sin. (b' - b^\circ)} \cdot \frac{\tau'}{\tau} + \frac{B}{r^3} - \frac{B}{R^3};$$

the quantities of the second order being omitted in the expression of ρ°; because ρ° is only to be used in valuing $\frac{B}{r^3}$ which is already of the second order.

If we substitute the values of the coordinates (No. 3), in the expressions $r^2 = x^2 + y^2 + z^2$; $r^3 = x^3 + y^3 + z^3$; $r'^3 = x'^3 + y'^3 + z'^3$; $V = xx' + yy' + zz'$; we shall get

* In this equation cos. b° must be written for its equal cos. λ° cos. $(\epsilon^\circ - n)$.

$r'^2 = R'^2 + 2R' \cos.\ \lambda' \cos.\ (e' - c') \cdot \varepsilon \cdot \rho + \varepsilon^2 \cdot \rho^2$

$r^2 = R^2 + 2R \cos.\ \lambda \cos.\ (e - c) \cdot \rho + \rho^2$

$r'^2 = R'^2 + 2R' \cos.\ \lambda' \cos.\ (e' - c') \cdot \rho' + \rho'^2$

$V = RR' \cos.\ (e' - e) + R \cos.\ \lambda' \cos.\ (e - c') \cdot \rho'$

$+ R' \cos.\ \lambda \cos.\ (e' - c) \cdot \rho + \cos.\ \gamma \rho \rho'.$

Cos. γ denoting here the same thing as before. The first of these formulas, which determines r'^2 when ρ is given, will enable us to compute ρ' when the value of ρ is known or assumed, by means of this formula found above, viz.

$$\rho' = \rho \times \frac{\sin.\ (b' - b)}{\sin.\ (b' - b'')} \cdot \frac{\tau'}{\tau} + \frac{B}{r'^3} - \frac{B}{R'^3}.$$

Thus the values of the functions, r^3, r'^3 and V will depend only upon one unknown quantity, namely ρ; which may therefore be found by the help of the same final equation as in the former method.

The preceding analysis leads us to the following method for determining the orbit of a comet, viz.

1st. We must begin with computing the values of θ, n, and i, as in the former method. We must then calculate

$$\cos.\ h = \cos.\ \lambda \cos.\ (c - n)$$

$$\cos.\ h' = \cos.\ \lambda' \cos.\ (c' - n)$$

$$\varepsilon = \frac{\sin.\ (b' - b)}{\sin.\ (b' - b'')} \cdot \frac{\tau'}{\tau + \tau'}$$

$$B = \frac{\tau'(\tau + \tau')}{2} \cdot \frac{\cos.\ (e'' - n) \sin.\ b'' - \cos.\ i \sin.\ (e'' - n) \cos.\ b''}{\sin.\ (b' - b'')} \cdot R''$$

$$\cos.\ \gamma = \cos.\ \lambda \cos.\ \lambda' \cos.\ (c' - c) + \sin.\ \lambda \sin.\ \lambda'.$$

2dly. We must reduce into numbers the several coefficients of the formulas, viz.

$$r'^2 = R'^2 + 2R'' \cos.\ \lambda'' \cos.\ (e'' - c'') \cdot \varepsilon \times \rho + \varepsilon^2 \times \rho^2$$

$$\rho' = \rho \times \frac{\sin.\ (b'' - b)}{\sin.\ (b' - b'')} \cdot \frac{\tau'}{\tau} + \frac{B}{r'^3} - \frac{B}{R'^3}$$

$r^2 = R^2 + 2R \cos.\lambda \cos.(e - c) \cdot \varrho + \varrho^2$

$r'^2 = R'^2 + 2R' \cos.\lambda' \cos.(e' - c') \cdot \varrho' + \varrho'^2$

$V = RR' \cos.(e' - e) + R' \cos.\lambda \cos.(e' - c) \cdot \varrho$

$+ R \cos.\lambda' \cos.(e - c') \cdot \varrho' + \cos.\gamma \cdot \varrho\varrho'.$

3dly. We must employ the same final equation as in the last method, to determine ϱ, upon which the values of r^2, r'^2, and V all depend.

Application to the first Comet of 1805.

If we turn to the calculation of the orbit of this comet, by the former method it will appear that the intersection of the great circle which passes through the two extreme places of the comet with the ecliptic is very near the place of the earth at the middle observation; for the distance of these two points, or the arc $e^\circ - n$, amounts to no more than $2^\circ\ 21'\ 16''\frac{1}{2}$. This instance therefore is very near the limit when the former solution becomes indeterminate; which takes place when the arc $e^\circ - n$ is evanescent, or when the points abovementioned coincide. In these circumstances we can hardly expect that the orbit will be determined with much certainty: for, on account of the small divisor tan. $(e^\circ - n)$, the values of δ, and the angle ω, will suffer considerable variations when small changes are made in the observed places of the comet, more particularly in the mean place. This comet therefore furnishes a very proper example for applying the method we have just investigated.

Using the same observations as in the former calculation, we shall get,

$$\log. 4\delta^3 = 9.2341873$$
$$n = 34^\circ\ 58'\ 31''\frac{1}{2}$$

$$i = 28\ 25\ 49$$
$$h = 124\ 51\ 1$$
$$h^0 = 145\ 29\ 34$$
$$h' = 154\ 0\ 41$$
$$\log.\ \varepsilon = 0{\cdot}042397$$
$$\log.\ \beta = 8{\cdot}457824$$
$$\cos.\ \gamma = 9{\cdot}941140$$

And the principal formulas of the orbit will be

$$r^{03} = 0{\cdot}98399 - \text{num.}\,(0{\cdot}244576)\,.\,\varrho + \text{num.}\,(0{\cdot}084795)\,.\,\varrho^2$$
$$\varrho' = \text{num.}\,(0{\cdot}079127)\,\varrho + \text{num.}\,(8{\cdot}457824)\,.\,\tfrac{1}{r^{03}} - 0{\cdot}029865$$
$$r^3 = 0{\cdot}988192 - \text{num.}\,(0{\cdot}104392)\,.\,\varrho + \varrho^2$$
$$\varrho'^3 = 0{\cdot}981987 - \text{num.}\,(0{\cdot}226675)\,.\,\varrho' + \varrho'^3$$
$$V = 0{\cdot}963404 - \text{num.}\,(9{\cdot}683982)\,.\,\varrho$$
$$- \text{num.}\,(9{\cdot}967035)\,.\,\varrho' + \text{num.}\,(9{\cdot}941140)\,\varrho\varrho'.$$

In order to shorten calculation, I shall assume $\varrho = 0{\cdot}51$, which is very near the true value, as will appear from the former computation of this orbit: then

$$r^{03} = 0{\cdot}40450 \ldots \log.\ 9{\cdot}6069185$$
$$9{\cdot}8034592$$
$$\log.\ r^{03} = 9{\cdot}4103777$$
$$\log.\ \varrho' = 9{\cdot}841343$$
$$r^3 = 0{\cdot}599714$$
$$r'^3 = 0{\cdot}294039$$
$$V = 0{\cdot}382873$$
$$l^2 = r^2 + r'^2 - 2V = 0{\cdot}128005$$
$$a^2 = 2r^2 + 2r'^2 = 1{\cdot}787506 \ldots \log.\ 0{\cdot}2522476$$
$$\log.\ a = 0{\cdot}1261238$$
$$1 + \tfrac{1}{2}\cdot\left(\tfrac{r^2-r'^2}{a^2}\right)^2 + \tfrac{1}{12}\cdot\tfrac{b^2}{a^2} = 1{\cdot}02059$$
$$b^2 - \tfrac{4b^2}{a} \times 1{\cdot}02059 = -\,0{\cdot}002888.$$

I next put $\varrho = 0{\cdot}52$; then

$r^3 = 0{\cdot}39946 \ldots$ log. $9{\cdot}6014733$

$9{\cdot}8007366$

log. $r^3 = 9{\cdot}4022099$

log. $\varrho' = 9{\cdot}850088$

$r' = 0{\cdot}597298$

$r'^2 = 0{\cdot}290041$

$V = 0{\cdot}377429$

$b^2 = 0{\cdot}132481$

$a^2 = 1{\cdot}774678 \ldots$ log. $0{\cdot}2491196$

log. $a = 0{\cdot}1245598$

$1 + \frac{1}{2}\left(\frac{r^2 - r'^2}{a^2}\right) + \frac{1}{12} \cdot \frac{b^2}{a^2} = 1{\cdot}02121$

$b^2 - \frac{4b^2}{a} \times 1{\cdot}02121 = +\ 0{\cdot}001037.$

By comparing the two errors, we shall get $\varrho = 0{\cdot}5174$; which must be substituted; then

$r^3 = 0{\cdot}40075 \ldots$ log. $9{\cdot}6028735$

$9{\cdot}8014367$

log. $r^3 = 9{\cdot}4043102$

log. $\varrho' = 9{\cdot}847831$

$r' = 0{\cdot}597907$

$r'^2 = 0{\cdot}291041$

$V = 0{\cdot}378829$

$b^2 = 0{\cdot}131304$

$a^2 = 1{\cdot}777896 \ldots$ log. $0{\cdot}2498820$

log. $a = 0{\cdot}1249410.$

If we interpolate $1{\cdot}02059$ and $1{\cdot}02121$, which correspond

to the two preceding values of ϱ, we shall get 1·02106 for the present value of the same quantity; then

$$b^3 - \frac{4k^2}{a} \times 1{\cdot}02106 = 0.$$

If we calculate the orbit from the values of $\varrho, \varrho', r^2, r'^2$ and b^2 that have just been found, we shall get the following elements, viz.

Log. of perihelion dist. -	9·574910
Time of passage, -	Nov. 18·0262
Inclination - -	15° 55′ 47″
Place of the ascending node	345 2 8
Place of the perihelion -	148 55 7

These quantities come surprisingly near the corrected elements of LEGENDRE: and, without further calculation, we may presume that they will represent even the middle place of the comet, Oct. 30th, within a minute of a degree.

Although the method of this article has been investigated for the purpose of supplying the deficiency of the former method, it is nevertheless quite general, and will apply in all cases whatever: and perhaps of all the methods hitherto proposed, this is the one in which the results will be least affected with the errors of observation. Perhaps too we are warranted in drawing this conclusion from the contents of the preceding paper, viz. that a first approximation to the orbit of a comet may be deduced from three geocentric observations that shall come nearer the true elements than has usually been thought.

Fig. 1.

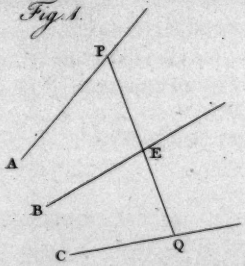


Fig. 2.

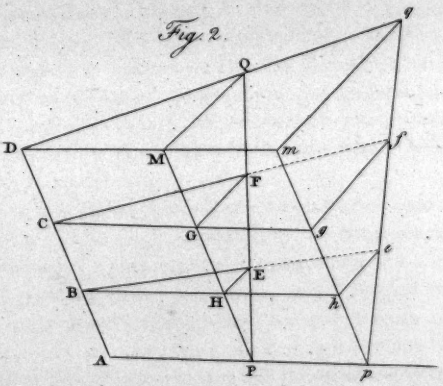
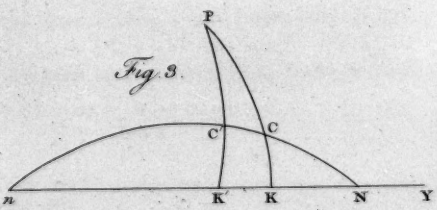


Fig. 3.



XI. *On the Affections of Light transmitted through crystallized Bodies.* By David Brewster, *LL.D. F. R. S. Edin. and F. S. A. Edin. In a Letter to Sir* Humphry Davy, *LL.D. F. R. S.*

Read December 23, 1813.

Dear Sir,

In a former paper* on "Some Properties of Light," which I took the liberty of addressing to you, and which the Royal Society honoured with a place in their Transactions, I attempted to give a brief abstract of a set of experiments on the Properties of transparent Bodies in refracting, dispersing, and polarising the rays of Light. An account of the instruments and methods employed in these experiments has since that time been published in my "Treatise on new philosophical Instruments."

From the general object of these researches, however, I have been allured into a new field of inquiry, by the discovery of a singular property of light transmitted through the agate, and the prosecution of the views which it suggested has led to some very extraordinary results, which, while they seem to conduct us into the very mysteries of physical optics, exhibit at the same time a series of appearances which far surpass, both in splendour and variety, all the phenomena of light under its usual transformations. In again soliciting you to communicate these observations to the Royal Society, I trust I need offer no apology. They are closely allied with that science which you have so widely extended by the most profound and brilliant

* Phil. Trans. for the year 1813, p. 101.

discoveries; and it is probably from the cultivation of this department of physics, that philosophy will be enabled to unfold the secrets of double refraction, to explain the forms and structure of crystallized bodies, and to develope the nature and properties of that ethereal matter, which, while it enlivens all nature by its presence, performs also a capital part in the operations of the material world.

The different subjects of which I mean to treat in the following letter may be included under five heads.

I. On the polarising power of the agate.

II. On the structure of the agate as connected with its optical properties.

III. On the peculiar colours exhibited by the agate.

IV. On the depolarisation of light.

V. On the elliptical coloured rings produced by obliquely depolarising crystals.

I. *On the polarising Power of the Agate.*

I have already shewn, in a former paper, that a ray of light transmitted through a plate of agate cut by planes perpendicular to the laminæ of which it is composed, suffers polarisation like one of the pencils formed by double refraction. If the light thus polarised is incident at a particular angle upon any transparent body, so that the plane of reflection is perpendicular to the laminæ of the agate, it will experience a total refraction; if it is transmitted through another plate of agate, having its laminæ at right angles to those of the plate by which the light is polarised, it will suffer total reflection; and if it is examined by a prism of Iceland crystal turned round in the hand of the observer, it will vanish and reappear in every quadrant of its circular motion.

The pencil of rays to which this remarkable property is communicated is surrounded by a large mass of nebulous light, which extends about 7° 30′ in length, and 1° 7′ in breadth on each side of the bright image.* This nebulous light never vanished with the bright image which it enclosed, but was obviously affected with its different changes, increasing in magnitude as the bright image diminished, and diminishing as the bright image regained its lustre. From this circumstance I was led to conjecture "that the structure of the agate was in "a state of approach to that particular kind of crystallisation "which affords double images, and that the nebulous light was "an imperfect image arising from that imperfection of struc-"ture."

On the supposition that this conjecture was well founded, I imagined, in conformity with the general analogy of all doubly refracting crystals, that the bright image and the nebulous light were produced by two different refractive powers, and I expected to separate the one from the other by forming the agate into a prism with a considerable refracting angle. Every attempt of this kind, however, was fruitless: no perceptible separation of the images was effected by any of the prisms which I employed, and I was therefore obliged to abandon this mode of investigation.

Having procured a plate of agate remarkably thin and transparent, I admitted a beam of light from the sky into a dark

* On each side of the bright image I have observed a condensation of the nebulous light resembling two imperfect images of the luminous body. These imperfect images, which increase in number by inclining the agate, are slightly tinged with the prismatic colours, which evidently belong to that class of phenomena which have been so ably treated by Dr. Thomas Young, in his late work on Medical Literature.

room through a narrow rectangular aperture. When this aperture was viewed through the agate, it was surrounded with a very considerable nebulosity; and by interposing a prism of Iceland spar between the agate and the eye, and giving it a motion of rotation, the nebulous light became very dense when the bright image vanished, and almost completely disappeared when the bright image had reached its greatest brilliancy. The bright and the nebulous images, therefore, comported themselves exactly like the two images formed by doubly refracting crystals; and the small portion of nebulous light, which surrounded the bright image at its maximum lustre, was obviously produced either by the imperfect polish of the agate, or by its not being cut exactly at right angles to the plane of its laminæ.

It will be seen from a subsequent section of this letter, that light polarised by the agate, or by any other means, is depolarised, or partly restored to its original state, by being transmitted in a particular direction through a plate of mica, or any other crystallized body. I therefore interposed a plate of mica between the agate and the Iceland spar when the nebulous light had nearly disappeared, and having adjusted it to the depolarising position, the nebulous light was instantly revived round the bright image, while the other bright image which had disappeared resumed its place in the middle of the other nebulous mass.

When a pencil of light polarised and afterwards depolarised, in a manner to be afterwards described, is transmitted through a plate of agate, the *red** rays go to the formation of the bright

* The *red* and the *green* are complementary to each other. The same result is obtained if the *blue* and *yellow*, or any other two complementary colours are used.

image, while the *green* rays compose the nebulous light, so that we have a *red bright image* enveloped in a *cloud* of *green light*. By turning round the agate 90° the bright image is formed by the green rays, while the nebulous image consists of the red rays, so that we have a *green bright image* encircled by a mass of *nebulous red light*. If in the place of the agate we substitute a doubly refracting crystal, it will always be found that the ordinary image is green when the extraordinary one is red, and that they assume these colours alternately during the motion of the prism round the axis of vision.

From these experiments, we may consider it as demonstrated, that the nebulous light has the same relation to the bright image, as the first has to the second image of all crystals that have the property of double refraction.* It does not appear, however, that the nebulous image is produced by a greater refractive power than that by which the bright image is formed. There is on the contrary every reason to conclude, in opposition to the analogy of all doubly refracting † crystals, that the agate gives two images and polarises them like other crystallized bodies, while the one image is placed exactly in the centre of the other.

II. *On the Structure of the Agate as connected with its optical properties.*

When we examine a piece of transparent and well polished agate, we perceive a number of bands or stripes, which are

* See Edinburgh Trans. Vol. VII. Part II.

† It will be seen from a subsequent paper, that many other bodies both of mineral, animal, and vegetable origin have the property of forming two images polarised in an opposite manner, but not produced by two different refractive powers.

the sections of a succession of laminæ that are sometimes parallel, but in general concentric. These laminæ are often of a milky white colour when seen by reflected light, and sometimes nearly as transparent and colourless as glass, and the white laminæ commonly alternate with the transparent ones. The laminæ which are white when seen by reflected light, are brown by transmitted light, and the intensity of this brown colour increases with the thickness of the plate of agate. The transparent laminæ exhibit three varieties of structure.

The *first variety*, which appears to be the coarsest, consists of a number of small serpentine lines like the figures 333333, lying parallel to each other, and closely resembling the surface of standing water when ruffled by a gentle breeze, or the sandy bottom of a slow moving stream. These serpentine lines are always arranged in a direction parallel to the laminæ, and are seen very distinctly even when the agate is so thin as the 150th part of an inch.

The *second* variety of structure differs from the first, only in the serpentine lines having a much smaller size; and the laminæ which have this structure appear the finest and most transparent.

The *third* variety has no serpentine lines, and does not appear to differ from other semi-transparent bodies. It admits the light more copiously in all directions than any of the other structures, and as it does not polarise it in a similar manner, we may consider it as possessing, in a different way, that kind of crystallisation which polarises the incident light by separating it into two pencils.

The white veins sometimes exhibit the first variety of structure, but in several specimens the veins appear to be fibrous

in their structure, the fibres stretching at right angles to the laminæ through the whole of their thickness.

These different structures will be better understood from fig. 1 and 2 of Plate V. Fig. 1, represents the specimen of agate with incurvated veins which I have noticed in a former paper.* It is composed of two veins, AB, CD, and three transparent portions AEB, ABDC, and CFD. The transparent portions exhibit the *second* variety of structure, though the small serpentine lines are not so distinctly marked as in other specimens. The two veins AB, CD, are both white when seen by reflected light. The breadth of AB is one-tenth of an inch and its radius of curvature $1\frac{1}{2}$ inch, and it consists of *four* smaller veins *mn*, *op*, *qr*, *wx*. The light reflected by *mn* is a paler white, and the light transmitted by it a lighter brown than in the other parts of the vein. The light which *op* reflects is of a brighter white, and that which it transmits of a deeper brown than in the other parts of the vein, and at the junction of *mn* and *op* there are several tufts of fibres of the same character as *op*. The other divisions of the vein *qr* and *wx* are of an intermediate character between *mn* and *op*. The vein CD resembles the division *mn*, and possesses, like AB, the fibrous structure already described. The thickness of the plate AEDC is one-fiftieth part of an inch; AC is three-tenths of an inch; and a line AC forms an angle of about 25° with a plane perpendicular to the laminæ.

Figure 2, represents another specimen of agate of a different character. It consists of transparent portions AB, BC, CD, DE, EF, separated from each other by white veins B*b*, C*c*, &c. and distinctly exhibiting the *second* variety of structure; and

* See Phil. Trans. 1813, Part I.

of other transparent portions FG, GH, HI, IK, KL, separated by similar veins Gg, H*h*, &c. but exhibiting the *first* variety of structure. All the veins possess a structure approaching to that of the first variety, Gg, K*k* exceeding the rest in the intensity of the light which they reflect and transmit.

If we measure the quantity of light transmitted through a plate of agate containing veins, it will be found to be a maximum when the direction of the incident rays is parallel to the interior surfaces of the veins. When the light, however, is transmitted through a part of the agate of an uniform transparency, and perfectly free from veins, the same result will be obtained, the intensity of the light being a maximum when its direction is parallel to that of the laminæ.

If AB, fig. 3, be a section of the specimen of agate represented in fig. 1, and *mn*, *op*, the direction of the laminæ inclined 25° to the surfaces of the plate, rays of light incident in the direction RS parallel to *mn* are more copiously transmitted than when they are incident in any other direction. When the pencil of light falls in the direction TV, its lustre suffers a great diminution: the light gradually assumes a red colour, and vanishes altogether when the obliquity is considerable. But if the pencil is incident at the same angle on the opposite side, as PQ, its lustre suffers very little diminution, and its colour is not sensibly altered.

These facts admit, to a certain extent, of an easy explanation if we suppose that the plate of agate consists of laminæ *mn*, *op* imperfectly transparent, alternating with laminæ *cd*, *ef*, which are more pervious to light, a structure which is indicated by the existence of a bright and a nebulous image. In this case the intensity of the light will obviously be a maximum

when the ray RS is parallel to *mn*, and a ray PQ will, within certain limits, suffer less diminution of lustre than a ray TV falling with the same angle of incidence on the other side of RS.

When RS and *mn* are perpendicular to the surfaces of AB, and when PQ and TV form equal angles with the perpendicular RS, their intensity should be equal; but this is by no means the case, for the transmitted light which is incident on the side T of RS appears to have a different character from that which is incident on the other side. We must therefore suppose that there is some other peculiarity of structure in the agate, connected probably with that particular kind of crystallisation which polarises light, to which this curious fact must be ascribed.*

The intensity of the light transmitted by the agate is likewise affected by its polarising property. If a ray R*r*, Plate V., fig. 4, is incident upon a piece of agate AB, so as to be polarised by reflection from the second surface, then, since it is polarised during its passage from *r* to E, the bright image will suffer total reflection at E, while the nebulous image will be transmitted like common light in the direction EG. If the agate is now turned round 90° the nebulous image will suffer total reflection at E, while the bright image will penetrate the second surface at E like common light. When the incident ray R*r* has different obliquities and the agate intermediate positions, the intensity of the transmitted light will be more or less affected by its polarising power.

The preceding observations on the laminated structure of

* See Edinburgh Transactions, Vol. VII. Part II.

the agate enable us to give a satisfactory explanation of so m singular appearances exhibited by that mineral.

In the specimen shewn in Plate V., fig. 5, the black lines represent the veins, and consequently the direction of the laminæ, and the dotted lines *ab*, *ac*, *cm*, *ck*, &c. are drawn through the vertices of the angles made by the veins; and consequently by the laminæ whenever they change their direction. When light is transmitted through a piece of agate of this description, the planes A*acm*, *mck*, *ngf*, *nhg* have the appearance of being differently inclined to one another, and transmit different quantities of light. If the veins and the laminæ preserved the same inclination to the surface of the plate of agate when they changed their direction at the lines *ac*, *cm*, *de*, the phenomenon which has been mentioned could not take place; but whenever the laminæ change their direction, their inclination to the surface likewise changes, and therefore the intensity of the transmitted light experiences a corresponding variation as the rays have to traverse different lengths of the imperfectly transparent laminæ.

When the veins and the laminæ are incurvated like those in the portion A*acm*, their inclination likewise changes, but as this change is gradual and not sudden, as in the former case, the intensity of the transmitted light suffers a gradual variation, and the portion A*acm* has the appearance of being concave. When the laminæ therefore are arranged in a circular form, they will resemble a number of dimples, the apparent concavity of which will in some cases depend on the curvature of the laminæ, and will exhibit the phenomenon of the *hammered agate*.

III. *On the peculiar Colours exhibited by the Agate.*

In my former letter on the polarising power of the agate, I noticed the existence of a coloured image which appeared on each side of the common colourless image, and which was polarised in a similar manner. I have since observed the same phenomenon in other specimens, and though I have not been able to discover its cause, I trust the following observations will be of some service to future inquirers.

In the specimen represented, in Plate V., fig. 2, the colours appear only when the rays of light are transmitted through the veins B, C, D, E, F, G, H, I, K, or through the coarse grained portions F*g*, G*h*, H*i*, I*k*, K*k*L, and when these parts are covered, no colour is perceived. If the eye, therefore, is placed behind any of the coarse grained parts, and close to the agate, a colourless image of a candle will be visible, and on each side of it a highly coloured image forming an angle of $10°\frac{1}{2}$ with the colourless image. The colours, which are extremely brilliant, are blue, green, yellow, and red, reckoning from the common image. A second image coloured in a similar manner, but considerably fainter, is distinctly seen forming with the colourless image an angle of about 21°. When the agate is held some inches distant from the eye, the colours appear diffused over the surface of the coarse grained portions, and when the light is strong, the phenomenon is uncommonly brilliant. When the vein B*b* is a pale blue, at a certain distance from the eye, C*c* and D*d* are of the same colour, E*e* is greenish, F*f* is yellowish, FG*gf* is pale red, and the red colour is more intense towards L*k*. By a gentle mo-

tion of the agate the colours of these portions instantly change, a particular colour being always produced in the same portions at a particular angle of incidence. The veins G*g*, H*h*, I*i*, and K*k* are, however, green when the surrounding portions are red, and red when the surrounding portions are green, from which it follows that these veins produce a particular colour at a different angle of incidence from the adjacent portions. In another specimen of agate, very like the preceding, the same phenomena are distinctly visible, and the coloured image forms the same angle with the common image. In a third specimen, belonging to ROBERT FERGUSON, Esq. of Raith, the colours are exhibited in the most splendid manner. A semitransparent and irregularly elliptical zone, about six and a half inches in circumference and three tenths of an inch broad, has the *first* variety of structure, and forms the coloured image at a distance of $13\frac{1}{2}°$ from the common image.

In the specimen represented in Plate V., fig. 1, the colours are visible only in the vein AB; but here the angle of the first coloured image with the common image is 28°, while that of the second image, which is very faint, is only a little greater. The other vein CD, which to all appearance has the same structure as AB, and which differs from it only in being a little thinner, exhibits no colours; but there is a small stripe *st* at its edge where the colours are very distinct. This circumstance induced me to think that the colours depended on the thickness of the plate, as well as upon its structure; but upon grinding a hollow place *mvw* in the vein AB, so as to make the agate remarkably thin, I found that it gave the same colours as before. A similar experiment was made with another piece of agate, and the result

was the same, though the thickness of the plate could not exceed the 400th part of an inch. The colours indeed were rendered more brilliant by the increased transparency of the agate, but in other respects they experienced no change. In another specimen, of which it is unnecessary to give a particular description, the coloured image formed an angle of about 34° with the colourless pencil, and the different veins produced the same colour at different angles of incidence.

In attempting to explain these appearances, I at first imagined that the colours arose from the polarisation of the transmitted rays, and that they were analogous to the colours of plates of mica and topaz which I have described in another place. I found, however, from several experiments, that the coloured image is equally distinct in every position of the agate; that it is alike produced by polarised or depolarised light, and that it suffers no change either when examined by a plate of agate or by a doubly refracting crystal.

The phenomenon which we have described must therefore be considered as a new case of the production of colour, and though we do not pretend to point out its cause, yet it obviously depends upon a particular structure which is possessed only by some portions of the agate, and admits of such variations as to produce the same colours at different angles of incidence.

IV. *On the depolarisation of Light.*

In the fourth book of my Treatise on new Philosophical Instruments, I have already shewn that almost all transparent

crystals possess in two positions the singular faculty of depolarising light, or of depriving it of the property which it acquires by transmission through the agate, while in other two positions of the depolarising crystal, the polarity of the light suffers no change. Thus in Plate V., fig. 5, let ABDC, be a piece of mica or of any other crystallized body interposed between a plate of agate and a prism of Iceland spar when one of the images has vanished, and let GH be parallel or perpendicular to the laminæ of the agate when the vanished image continues invisible. This line I have called the *neutral axis*, as no effect is here produced upon the polarised light. By turning the mica round, the vanished image will gradually appear, and when the line AD comes into a vertical position, it will be restored to its full lustre, and will never again vanish whatever be the position of the Iceland spar. The line AD I have therefore called the *depolarising axis*, as the light in passing through it has been deprived of the polarity communicated by the agate, and which prevented it from penetrating the rhomboid of Iceland spar.

By continuing the motion of the mica, it will be found that EF is also a neutral axis, and BC a depolarising axis. The depolarising axes are common to almost all crystallized substances, and what is very singular, I have discovered them in horn, gum Arabic, glue, tortoise-shell, caoutchouc, gold beater's skin, amber, mother of pearl, camphor, spermaceti melted and cooled, bees' wax melted and cooled, adipocire melted and cooled, manna, oil of mace, acetate of lead melted and cooled, human hair, bristles of a sow, human cornea, cornea of a fish, cornea of a cow, and imperfectly in some pieces of plate glass.

Plates of mica, however, while they possess the properties of all depolarising crystals, exhibit phenomena peculiar to themselves. If the neutral axis GH of a plate of mica is inclined forwards so as to make a considerable angle with the horizon, the image that was formerly invisible will start into existence, and therefore the neutral axis GH is accompanied with an oblique depolarising axis N*n*. This oblique axis is also possessed by topaz, rock crystal, and many other crystallized bodies. In making the same experiment with the depolarising axis of the mica, I observed the image to vanish in the direction M*m* and P*p*, which I considered as oblique neutral axes, but I have since found that this was owing to the polarisation of the pencil by oblique transmission, a property of light which I had not then discovered.

We have hitherto considered the depolarisation of light as effected by two separate bodies, one of which polarises the incident rays, while the other deprives them of the polarity which they have thus acquired; but in all bodies that possess oblique depolarising axes, light may be polarised and depolarised by the same crystal. Thus if AB*ab*, Plate V., fig. 7, be a plate of topaz having DE for its oblique depolarising axis, and if a ray RR′ of common light is incident at R′ with such an obliquity that it is polarised by being reflected at C from the posterior surface *ab*, then the ray *rr′* will be depolarised in its passage from C to *r* along the oblique axis of depolarisation, and the emergent ray *rr′* will be depolarised light. Hence it follows that the angle DC*b*, which the oblique depolarising axis makes with the posterior surface *ab*, is nearly equal to the

complement of the angle OC*r*, at which light is polarised by reflection at C.*

V. *On the elliptical coloured rings produced by depolarising Crystals.*

In a former work, to which I have already had occasion to refer, I have given some account of the colours which accompany the depolarisation of light, and I have particularly noticed the remarkable fact, that when a beam of white depolarised light is transmitted through a doubly refracting crystal, the red rays go to the formation of one image, while the bluish green rays go to the formation of the other image. In repeating and extending these experiments, I have been led into a new field of inquiry which has already afforded a series of instructive results deduced from a class of phenomena unquestionably the most brilliant within the whole range of optics.

The plate of topaz which was used in these experiments, is

* Since the preceding section was written, I have performed a very extensive series of experiments on the depolarisation of light, and have thus been led to a satisfactory generalisation of the phenomena. In this theory the phenomena are referred to the general principle of polarisation: such bodies as have neutral and depolarising axes are supposed to form two images polarised in an opposite manner, and either produced by the same or by different refractive powers; while those which depolarise light in every direction, like gum Arabic, caoutchouc, &c. are composed of films or layers, each of which is a doubly polarising crystal, the neutral and the depolarising axes of one film not being coincident with the neutral and depolarising axes of the rest. In a separate memoir, which I have drawn up for the consideration of the Royal Society, I have given a full account of this theory, of the experiments on which it is founded, and of the new views to which it leads respecting the formation and structure of organised matter.

about $\frac{103}{1000}$ of an inch thick, and has two natural faces which are parallel and highly polished. Its refractive power is 1.636; its dispersive power 0.024, and the angle at which it polarises light by reflection 58° 8′. It is represented in section by AB*ab*, in Plate V., fig. 8, DE being one of its depolarising axes. If a beam of *common light* RR′ is now incident on the anterior surface AB at an angle of about 60° 38′, a part of the beam will penetrate the topaz at R, and after reaching the posterior surface *ab*, it will be partly transmitted at C in the direction CF, and partly reflected in the direction C*r*, so as to depart from the point C almost wholly polarised by reflection; but in its passage from C to *r* along the oblique depolarising axis of the crystal, it is depolarised and emerges at *r*, in the direction *rr′* deprived of the polarity which it had acquired by reflection at C. If the observer now looks into the topaz in the direction *r′r*, through a plate of agate having its laminæ perpendicular to the plane of the section AB*ba*, he will perceive about ten brilliantly coloured elliptical rings, four of which, with the two central spots, are shewn in Plate VI., fig. 1.*

The following measures will convey a correct idea of their form and magnitude.

Breadth of the central spots including half the black space between them - - - -	1° 51′
Distance of the outsides of the central spots -	3 42
Transverse length of each central spot - -	5 7
Extreme conjugate diameter of *first* red ring -	7 24

* I have counted fourteen of these rings when the light was polarised by oblique transmission through a plate of mica 0.127th of an inch thick. The colours are in this case much more distinct.

Extreme conjugate diameter of second red ring	-	11°	6′
Ditto	third - -	14	48
Ditto	fourth - -	18	30
Ditto	fifth - -	22	12
Ditto	sixth - -	25	54
Ditto	seventh - -	29	36
Ditto	eighth - -	33	18
Ditto	ninth - -	37	0
Ditto	tenth -	40	42
Black space between the oval centres	- -		14½

In order to convey a correct notion of the different colours which compose the elliptical rings, and which vary in different parts of the same ring, I have given in Plate VII., fig. 1, an outline of the first *six* rings with references to the following table, which contains the colours in five different parts of the semicircumference of each ring.

Oval central spots.
1. *Light blue* with a purplish tinge fading into *white* above, and gradually deepening into *black* below.
2. *White* fading into yellow above, and *light blue* below.
3. *Yellow* shading off into *white* below, and red above.
4. Red, with a pink tinge, and shading into *yellow* below.

I. Order.
5. *Black* fading into light blue towards 6.
6. *Light blue* fading into *green*.
7. *Yellow* shading into *red*.
8. Deep crimson.

II. Order.
9. *Blue*, very little.
10. *Green*, very little, the green beginning a little below.
11. *Yellow* shading into red.
12. Crimson.

III. Order.
13. Light blue, very little.
14. Green, very broad.
15. Crimson fainter than 12.

IV. Order.
16. Green, very broad.
17. Faint blue.
18. Faint crimson.

V. Order.
19. Very faint blue.
20. Very faint crimson.

VI. Order.
21. Still fainter blue.
22. Still fainter crimson.

I. Order.
- 23. Black shading off into light blue.
- 24. Dark green.
- 25. Yellow.
- 26. Deep crimson.

II. Order.
- 27. Blue.
- 28. Green.
- 29. Crimson.

III. Ord.
- 30. Bluish green.
- 31. Crimson.

IV. Ord.
- 32. Very faint blue. Green begins here.
- 33. Very faint pink.

V. Order.
- 34. Still fainter blue.
- 35. Still fainter crimson.

I. Order.
- 36. Black mixed with a little green.
- 37. Ditto.
- 38. Ditto.
- 39. Dark green, a little yellow on its upper side.
- 40. Red, pinkish, not very bright.

II. Order.
- 41. Darkish green.
- 42. Faint crimson.

III Order.
- 43. Green.
- 44. Blue, very little.
- 45. Faint crimson.

IV. Order.
- 46. Faint blue.
- 47. Faint crimson.

All the other fringes, without this, consist of blue and pink, which grow fainter as they recede from the centre.

I. Order.
- *a* Black.
- *b* Dark blue approaching to black.
- *c* Light blue shading to black.
- *d* Whitish.
- *e* Reddish brown of an orange cast.

II. Order.
- *f* Very dark blue.
- *g* Light blue.
- *h* Yellowish.
- *i* Pink.

III. Ord.
- *k* Light blue.
- *l* Yellowish green.
- *m* Faint pink.

IV. Ord.
- *n* Green, blue begins here and runs downward.
- *o* Pink.

V. Order.
- *p* Faint blue, green begins here and runs downward.
- *q* Pink.

I. Order.
- *r* Black.
- *s* Faint blue shading into whitish.
- *t* Whitish shading into faint brown.
- *u* Faint reddish brown.

II. Order.
- *v* Black.
- *w* Dark blue shading into light blue.
- *x* Brownish yellow.
- *y* Dark pink, with a brown tinge.

III. Order.
- *z* Pale blue shading into greenish yellow.
- *&* Greenish yellow.
- *a'* Pink.

IV. Order.
- *b'* Blue, not much.
- *c'* Green.
- *d'* Pink.

V. Order.
- *e'* Green.
- *f'* Light blue.
- *g'* Faint pink.

VI. Order.
- *h'* Faint blue.
- *i'* Faint pink.

If the plate of agate is now turned round 90°, so that its laminæ are parallel to the plane of the section AB*ab*, a *second set* of elliptical rings will be seen as represented in Plate VI., fig. 2, which is on the same scale as fig. 1, and which contains only the four first orders of colours, and the central spots. This new set of rings is composed of colours which are *complementary* to those in the first set. By measuring the diameters of the *red* rings in the *second set*, it will be found that they correspond with those of the *green* rings in the *first set*; the *blue* rings correspond with the *yellow*; the *green* with the *red*; and the *yellow* with the *blue*; and in the outer rings the *blue* with the *pink*, and the *pink* with the *blue*. The central spots in the *second set* exhibit the same opposition of colours to those in the *first set*; but they are smaller, and placed at a greater distance; and the space around them which was formerly *black* is now *white*.

If instead of a plate of agate we employ a doubly refracting crystal, the *first set* of rings will, in one position of the crystal, be seen in the first image; and upon turning the crystal about its axis, the *first set* will occupy the second image, and the *second set* the first image, an alternation taking place in every quadrant of the motion of the crystal. This method of viewing the rings is in some respects superior to that in which the agate is used, as the nebulous image formed by this mineral injures, in some degree, the distinctness of the image; but on the other hand, the doubly refracting crystal requires to be cut into a prism with a large angle, in order to separate the two images which it forms, and therefore it alters the shape of the rings, and produces a complete change upon their colours.*

* Since this paper was written, I have discovered a new property of light in virtue

If the emergent rays *rr'*, instead of being transmitted through agate or Iceland spar, are reflected at the polarising angle from any transparent body having its reflecting surface parallel to the plane of the section AB*ab*, they will exhibit the *first set* of rings; but if the reflecting surface is perpendicular to the plane of the section, the *second set* of rings will be visible. When the *first set*, thus seen by reflection, is examined through a prism of Iceland spar, it suffers no change either in the first or second image.

In these experiments the *first set* of rings is extremely distinct, as the polarising crystal extinguishes the light RS reflected from the first surface of the topaz; but the *second set* of rings is very faint, as the light RS is not extinguished by the polarising body.

When we examine the transmitted light CF, either with the naked eye or with polarising crystals, no coloured fringes are visible.

Such are the modifications which *common light* undergoes in its passage through topaz. The affections of *polarised light*, which now come under consideration, are still more varied and interesting. In my first experiments on this subject, I polarised the light by transmitting it through the agate; but I afterwards found it most convenient to communicate this property by reflection from the surface of a transparent body.

Let RR', Plate V., fig. 9, be a beam of polarised light obtained by reflection from any transparent body GH, the plane

of which it is polarised by oblique transmission through transparent bodies. Hence, in all my experiments on the coloured rings, I find it of incalculable advantage to polarise the light by bundles of glass plates, and to use them in every case where I formerly employed agate or calcareous spar.

of reflection from GH being perpendicular to the plane of reflection from the topaz AB. A part of this beam will be reflected at C in the direction C*r*, and part of it transmitted at C in the direction CF, no light being reflected from the first surface AB. The rays transmitted at C having been polarised before their incidence at R′ are depolarised in passing from R′ to C along the oblique depolarising axis, and the rays reflected at C are polarised by reflection from the surface *ab*, and again depolarised in their passage from C to *r* along the other oblique depolarising axis.

If the observer now looks into the topaz in the direction *rr′*, he will perceive the *first* set of elliptical coloured rings, as represented in Plate VI., fig. 1. These rings are now peculiarly distinct and brilliant, and it was therefore from them that I drew up the table of colours referred to from Plate VII., fig. 1.

Let the ray *rr′* be now received upon a plate of agate having its laminæ perpendicular to the section AB*ab*, and a *third set* of rings will be seen like those in Plate VI., fig. 3. This *third set* differs from the *first set* only in the central parts. All the rings have the same colours in both, but the central spots are much smaller in the third set than in the first, and the mass of darkness with which they are surrounded encroaches considerably upon the blue part of the first ring.

In the third set of rings the distance of the outsides of the two central spots is - - 3° 3′

Conjugate diameter of each spot - - 1 1

Ditto of the black space between the spots - - - - 1 0

The *third set*, indeed, may be considered as the exact counterpart of the *second* set, all the colours of the former being

complementary to those of the latter, and the central spots having the same form and magnitude.

If the plate of agate is now turned round, so that its laminæ are parallel to the section AB*ab*, a *fourth set* of rings will be seen. This new set, which is represented in Plate VI., fig. 4, is by no means brilliant, but it is distinguished from all the rest by striking peculiarities. In its general structure it resembles the *first set*, but in the middle of each central spot there is a darker spot composed of blue and red, with a little green above the blue, and every ring is divided into two rings, each of which has the same colours as the original ring. This division of the rings occupies only a part of the semicircumference of each, and is not seen beyond the third ring.

When the agate begins to move from the position which gives the *third set* of rings, into that which gives the *fourth set*, two blue spots and the divisions of the rings begin to appear at $a, a, a, a, a', a', a', a'$, Plate VII., fig. 2, and move along the lines abc, $a'b'c'$ till they arrive at $c, c, c, c, c', c', c', c'$, when the rings assume the appearance of the *fourth set*. If the agate performs another revolution of 90° from the position which gives the *fourth set* into that which gives the *third set*, the blue spots, and the divisions of the rings move off in the direction c, d, e, c', d', e' till the rings assume the appearance of the *third set*.

The phenomena which have now been described may also be perceived, when the polarisation of the rays rr' is effected either by a doubly refracting crystal or by reflection. In one position of the doubly refracting crystal the third set of rings is seen in the first image, and the fourth set in the second image, and they alternate in every quadrant of the motion of the crystal. When the ray rr' is reflected from a transparent

body, so that the plane of reflection is parallel to the plane of reflection from the topaz, the *fourth set* of rings will be visible.

Hitherto we have attended only to the light reflected from *ab*, the posterior surface of the topaz; but the light transmitted at C exhibits also interesting phenomena. When the observer looks through the topaz in the direction FC, so as to see the polarising surface GH, the *second set* of rings is faintly visible. They become extremely distinct, however, when viewed through a plate of agate having its laminæ at right angles to the plane AB*ab*. If the laminæ are parallel to the plane AB*ab*, the *second set* is converted into the *first set* with colours a little paler than when it was produced in the former experiments.

In the preceding experiments the plane of reflection from GH has been perpendicular to the plane of reflection from the topaz. We shall now describe the phenomena which take place when these planes are parallel to each other, an arrangement which is represented in Plate V., fig. 10.

When the observer looks into the topaz in the direction rr', he will perceive the *second set* of fringes. If the rays rr' are transmitted through a plate of agate having its laminæ perpendicular to the plane of reflection, the *fourth set* of fringes will be seen, but they are very much fainter than they appeared in the former experiments. When the laminæ of the agate are parallel to the plane of reflection, the *second set* is faintly visible. The central spots are, however, rather larger than before, so that this set has the appearance of being the reverse of the *first* rather than of the *third* set.

When the light transmitted in the direction CF is seen by the naked eye, it exhibits the *first set* of rings. If it is examined through a plate of agate having its veins perpendicular to the

plane of reflection, the *first set* is still visible; but when the agate is turned round 90° the *second set* is perceived.

All the preceding observations were made with a plate of topaz $\frac{103}{1000}$ of an inch thick. When the plate has a greater thickness the rings are much smaller, and when it has a less thickness the rings are extremely large, so that in very thin plates, only a small portion of a ring can be perceived at once. We have already seen that with a plate $\frac{103}{1000}$ of an inch thick, the fourth red ring subtends an angle of 18° 30′. With another plate $\frac{227}{1000}$ of an inch thick, the angle subtended by the same ring is 8° 24′. But since

$$\frac{227}{1000} : \frac{103}{1000} = 18° 30' : 8° 24';$$

it follows *that the conjugate diameters of the rings are inversely as the thickness of the plates.*

According to the Abbé Hauy, the angle formed by two of the primitive faces of the topaz is 124° 22′; and therefore the axes or longest diagonals of the primitive rectangular prism will form angles of 60° 31′ 15″ * with a line perpendicular to the laminæ, a result which agrees so nearly with 60° 38′, the inclination of the axes of the coloured rings, that we can have no hesitation in concluding that *the axes of the coloured rings are coincident with the longest diagonals of the primitive rectangular prism.*

The rings which have now been described as produced by *topaz*, I have discovered in *rock crystal*, *mica*, the *agate*, the *oriental ruby*, the *emerald*, *native hydrate of magnesia*, *amber*, *ice*, *sulphate of potash*, *tartrate of potash and soda*, *nitrate of potash*, *acetate of lead*, *acetate of lead melted and cooled*, *prussiate of pot-*

* According to my own measurements the angle is 123° 58′, which gives 60° 28′ 26′ for the inclination of the diagonals.

ash, mother of pearl, bones of a cod, quill, the human nail, horn, tortoise shell, cornea of a fish, cornea of a cow, cornea of a man, spermaceti, RUPERT'S *drops, gum Arabic,* and *caoutchouc.*

1. *Rock crystal.* The only specimen of this mineral which I could obtain when I made the preceding experiments, was in the form of a double convex lens about $\frac{61}{1000}$ of an inch thick. It exhibited only segments of the coloured rings, but they were very large and brilliant, and afforded me the means of making a very interesting experiment with a plate of agate.

If a beam of common light is incident upon the neutral axis of this crystal, at such an angle that after reflection from its posterior surface, it shall emerge in the direction of its oblique depolarising axis, the light thus polarised by reflection and depolarised by transmission through the depolarising axis will reach the eye in the state of white light. If this light is viewed through agate, one of the coloured segments, suppose *green*, will be distinctly visible; but if the agate is turned 90° round, the green colour will be converted into *red*, and in general the colour seen in one position of the agate will be complementary to that which is seen in the other position. When the light, however, is brilliant, another very singular phenomenon presents itself. If the bright image seen through the agate is *green*, the nebulous image, in which it is inclosed, will be *red*; and when the bright image is red, the nebulous image will be green, and in general the colour of the nebulous image will be always complementary to that of the bright image. If we employ a prism of Iceland spar to examine the depolarised light, the colour of the ordinary image is always complementary to that of the extraordinary image. We may therefore consider the preceding result as an *experimentum crucis*, which

establishes the opinion respecting the structure of the agate, that has been given in another part of this paper.

2. *Mica.* The coloured rings are distinctly visible in mica, both when the light is transmitted perpendicularly through the plate, and when it is incident in the direction of its oblique depolarising axis. The irregular structure of this mineral, however, and the impossibility of procuring laminæ with parallel and even surfaces prevented me from investigating the phenomena of its coloured rings.

3. *Agate.* The only plate of agate in which I have observed the coloured rings, is cut in such a direction that it does not polarise the bright image. It possesses, however, the faculty of depolarisation, and therefore must form two bright images one of which lies immediately above the other. This plate is about $\frac{38}{1000}$ of an inch thick, contains no veins, and exhibits broad segments of coloured rings.

4. *Oriental ruby.* This doubly refracting crystal affords beautiful rings, in which, owing to the colour of the mineral, the predominant colours are crimson, light blue, and bluish green. The central spots were distinctly visible, and though the crystal was $\frac{13}{100}$ of an inch thick, the rings appeared to be larger than those given by topaz $\frac{103}{1000}$ of an inch thick.

5. *Emerald.* The coloured rings formed by this stone are principally blue and greenish yellow, the least refrangible rays being extinguished by the green colour of the mineral.

6. *Native hydrate of magnesia.* This mineral affords very distinct segments of coloured rings when the light is transmitted in a direction nearly perpendicular to the surface of the laminæ. Owing to the imperfect structure of the plates, I could not obtain a measure of the diameter of the rings.

7. *Amber.* As this substance possesses no crystalline form, and does not split into laminæ, I found it impossible to make any satisfactory experiments with it. The enormous breadth of its coloured rings was conspicuous in every specimen, but though I ground and polished more than twenty plates of it, I could not obtain one which exhibited any thing more than broad coloured segments. With a parallelopiped of amber 0.566 of an inch long, 0.300 broad, and 0.367 deep, the coloured segments were visible in every direction in which the light was transmitted. They appeared most distinct through the thickness 0.367; and through the thickness 0.566 they were still so broad, that no more than one colour of each ring could be seen. In a piece of amber $\frac{4}{100}$ of an inch thick, the rings were broader than in a plate of topaz $\frac{4}{300}$ of an inch thick.

8. *Ice.* The difficulty of making experiments upon ice without melting it, the want of a crystalline form, and the impracticability of shaping it into parallel plates prevented me from obtaining any accurate results. The following experiments, however, will throw some light upon this subject.

A piece of ice $\frac{1}{100}$ of an inch thick gave rings much broader than those exhibited by a plate of topaz $\frac{1}{300}$ of an inch thick. The rings were also seen by the reflection of common light from the posterior surface of the ice, the light reflected from the anterior surface being extinguished by a prism of calcareous spar.

A piece of ice $\frac{10}{300}$ of an inch thick exhibited rings larger than those given by a plate of topaz $\frac{3}{300}$ of an inch thick. The breadth of one of the fringes shewn by a plate of ice $\frac{24}{3000}$ of an inch thick was nearly 5° 26′, which compared with the

results already mentioned, gives $\frac{10.5}{300}$ for the thickness of a plate of topaz that would produce a fringe of the same magnitude. Hence the thicknesses of ice and topaz that give rings of equal size are as $\frac{24}{300}$ to $\frac{10.5}{300}$, or as 8.95 to 1, which is nearly the inverse ratio of $(m-1)^3$ in ice to $(m'-1)^3$ in topaz, m and m' being the indices of refraction. If we take $m = 1.307$ and $m' = 1.636$, this ratio will be nearly as 8.9 to 1. In these experiments the two oval central spots were distinctly seen.

Light transmitted at an angle of 46° through a plate of ice 1.25 inches thick gives rings of the same size as when it is transmitted at an angle of 60° 38′ through a plate of topaz $\frac{103}{1000}$ of an inch thick. By calculating the real thicknesses in the direction of the transmitted light, it will be found that the thicknesses at which ice and topaz produce rings of the same magnitude are as 8.4 to 1, a ratio not very remote from that of $(m-1)^3$ in ice to $(m'-1)^3$ in topaz.

Light transmitted at an incidence of 36° through a plate of ice $\frac{643}{1000}$ of an inch thick gave rings twice as large as those shewn by a plate of topaz $\frac{103}{1000}$ of an inch thick. These thicknesses will be found, after reduction, to be as 8.2 to 1, a ratio more remote than any of the former from that of $(m-1)^3$ to $(m'-1)^3$.

A plate of ice taken from the surface of a pool of water did not appear to depolarise light, when it was incident perpendicularly: but when the angle of incidence was considerable, the light was depolarised in every direction, and the coloured rings appeared even at great obliquities.

9. *Sulphate of potash.* A plate of sulphate of potash $\frac{135}{1000}$ of an inch thick gave fringes of colour, each of which was 4° in breadth, while another plate $\frac{163}{1000}$ of an inch thick gave

fringes 8° 12′ in breadth. Now

$$135 : 168 = 8° 12′ : 4° \text{ nearly},$$

so that the diameters of the rings are inversely as the thicknesses of the plates, as in the case of topaz. The light was incident on the sulphate of potash at an angle of 41°, which gives $\frac{179}{1000}$ for the oblique thickness of the plate $\frac{135}{1000}$ of an inch thick. Now

$$\frac{179}{1000} : 1° 51′ = \frac{309}{1000} : 2° 2′,$$

the size of the ring that would have been produced by a plate of topaz $\frac{179}{1000}$ of an inch thick, so that the thicknesses of sulphate of potash and topaz that produce equal rings are as 1.85 to 1, which is not very far from the ratio of $(m-1)^3$ to $(m'-1)^3$. If we take $m = 1.509$ and $m' = 1.636$ this ratio will be as 1.95 to 1.

10. *Tartrate of potash and soda.* The neutral axes of this salt are parallel and perpendicular to the axis of the prism, and it possesses an oblique depolarising axis along which the coloured rings are visible. The thicknesses of this substance and of topaz, at which equal rings are produced, are as 31 : 16, which is almost exactly the ratio of $(m-1)^3$ to $(m'-1)^3$. The value of m' in the tartrate being 1.515.

11. *Nitrate of potash.* This salt, which is remarkable for its optical properties,* exhibits along the axis of the hexaedral prism a series of beautiful miniature rings, *twelve* of which are distinctly visible. In a plate of the nitrate of potash $\frac{17}{100}$ of an inch thick, the fourth ring subtended an angle of 5° 45′, whereas, in a plate of topaz $\frac{461}{1000}$ of an inch thick, it subtended an angle of 8° 25′. But

* I have endeavoured to give a full account of these in the Transactions of the Royal Society of Edinburgh, Vol. VII. Part II.

$5° 45' : 8° 25' = \frac{461}{1000} : \frac{675}{1000}$

the thickness of topaz that would give the fourth ring a diameter of $5° 45'$. Hence the thicknesses at which the nitrate of potash and topaz produce rings of equal magnitude are as $\frac{170}{1000}$ to $\frac{675}{1000}$, or as 1 to 3.97 nearly. But assuming the rings to vary as $(m-1)^3$, those formed by the nitrate should have been larger than those exhibited by the topaz in the ratio of $686^3 : 515^3$, or nearly 1.88 to 1. Hence the rings formed by nitrate of potash are 1.88×3.97, or 7.5 times smaller than they should be if their conjugate diameters had varied as $(m-1)^3$.*

12. *Acetate of lead.* This doubly refracting crystal melts at a temperature not much greater than that which bees' wax requires, and it takes a long time to cool and crystallize. When it is formed by heat into a thin film between two plates, the crystals shoot from different centres, and exhibit by polarised light the most beautiful alternations of the prismatic colours. When the eye is kept at a distance from the plate, the colours radiate like the spicula of the salt, and vary at every inclination of the plate.

13. *Mother of pearl.* The coloured rings are extremely brilliant in this substance when the polarised light is transmitted almost perpendicularly; but they do not appear when it penetrates by an oblique path.

The other substances, which have already been mentioned as affording coloured rings by polarised light, exhibit only imperfect segments of the fringes, but in all of them these

* The thickness of the plates of ice, sulphate of potash, and nitrate of potash, and the inclination of the incident pencil were measured in the rudest manner, as my object was merely to ascertain in general if there was any connection between the magnitude of the coloured rings and the refractive power of the body which produced them.

segments are distinctly visible, excepting in caoutchouc, where the colours are extremely faint.

It is highly probable that the coloured rings will be found in a still greater number of crystallized bodies. I have sought for them in vain in the diamond, native orpiment, Iceland spar, fluor spar, muriate of soda, carbonate of lead, carbonate of barytes, the sclerotic coat of the eye, the crystalline lens, and a great variety of other bodies, and in some of these with so much care, that they could scarcely have escaped my notice, if they did exist. It therefore still remains to be determined, what kind of crystallization is necessary to their production, and what relation exists between the magnitude of the rings and the refractive power of the body which produces them. In some of the experiments already described, the diameters of the rings seem to vary as $(m-1)^3$, but the anomalies exhibited by amber and nitrate of potash completely prove that this is not the law by which their magnitude is regulated.

I have the honour to be,
Dear Sir,
your most obedient humble servant,

DAVID BREWSTER.

To Sir HUMPHRY DAVY, LL. D., &c. &c.

Fig. 1.

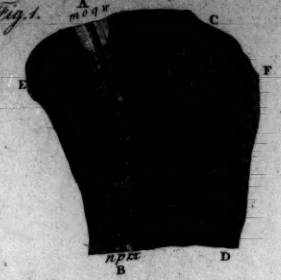


Fig. 2.

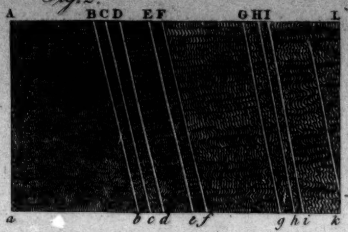


Fig. 3.

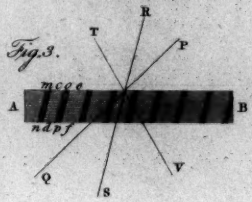


Fig. 4.

Fig. 5.

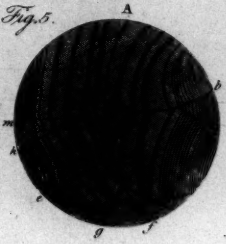


Fig. 6.

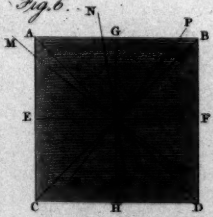
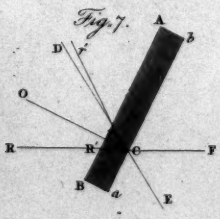


Fig. 7.



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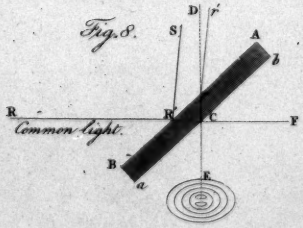


Fig. 9.

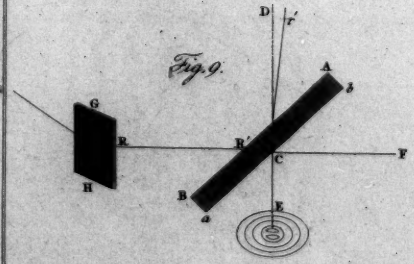
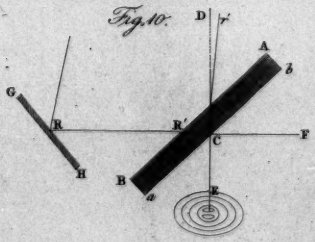


Fig. 10.



First Set. Fig. 1.

Fig. 3 Third

Second Set. Fig. 2.

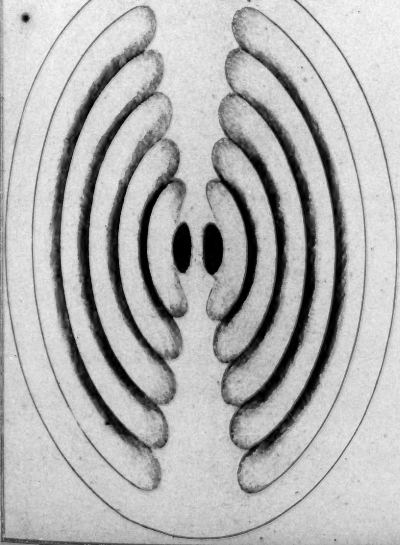
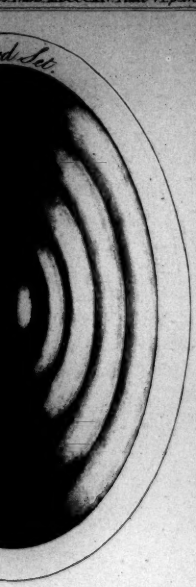


Fig. 4 Fourth


3d Set.

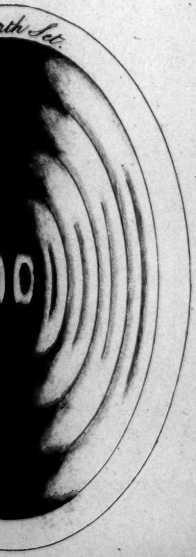

4th Set.

Fig. 1.

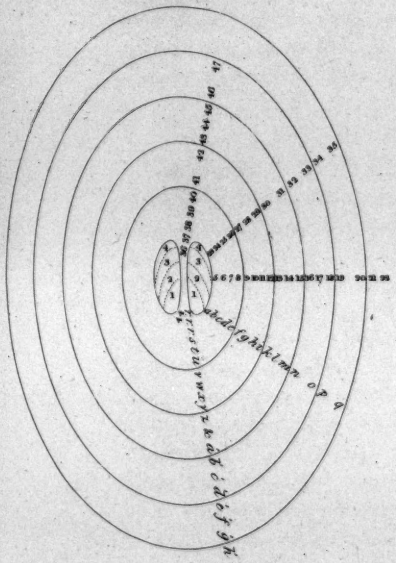
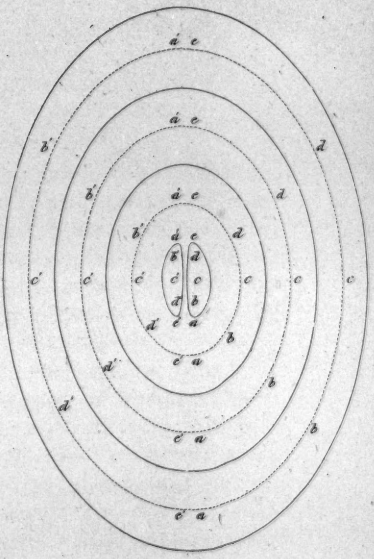


Fig. 2.



XII. *On the Polarisation of Light by oblique transmission through all Bodies, whether crystallized or uncrystallized. By* David Brewster, *LL.D. F.R.S. Edin. and F.S.A. Edin. In a Letter addressed to* Taylor Combe, *Esq. Sec. R.S.*

Read January 27, 1814.

Sir,

In a letter on "the Affections of Light in its passage through crystallized Bodies," which I had the honour of transmitting a few days ago to the Royal Society through Sir Humphry Davy, I alluded to a series of experiments which I had in view for the purpose of generalising the various phenomena which had been described. At the very commencement of this enquiry I have been led to the important general result "that "light transmitted obliquely through all transparent bodies, "whether crystallized or uncrystallized, suffers polarisation "like one of the pencils formed by doubly refracting crystals," and I hasten to communicate to you a brief sketch of the nature and consequences of this discovery.

In examining if any change was produced upon common light during its passage along the oblique depolarising axis of mica, I observed, in one position of the mineral, some appearances which indicated a partial polarisation of the incident rays. Upon turning the mica round, so as to preserve its obliquity to the incident pencil, the same phenomena presented themselves in every part of the revolution of the mica, and the quantity of polarised light was found to increase with the

obliquity of its incidence. I then substituted a plate of glass instead of the mica, and a similar result was obtained, though the quantity of polarised light was considerably less than in the first experiment. By adding one plate of glass after another, the number of polarised rays was increased by the addition of each plate, and when the plates amounted to *fifteen*, the transmitted pencil was wholly polarised at an angle of about 70° 17′, and possessed all the properties of that species of light.

When a beam of light, polarised in this manner, is viewed through a piece of agate having its laminæ parallel to the plane of refraction, the bright image of the object from which the ray proceeds will vanish, and the nebulous light will be a maximum; but if either the plates of glass or the agate is turned round, so as to bring the laminæ perpendicular to the plane of refraction, the nebulous light will vanish, and the bright image will recover its full lustre.

If we now substitute in place of the agate another series of plates, having their refracting surfaces parallel to those of the first series, and having the same inclination to the emergent ray as the first series had to the incident ray, the ray emerging from the first series will be transmitted through the second series, like ordinary light; but if the refracting surfaces are perpendicular to each other, the ray emerging from the first series will refuse to penetrate the second series, and the object from which it proceeds will be invisible.

Having thus ascertained that the pencil of light polarised by transmission comported itself, in every respect, like one of the pencils formed by double refraction, my next object was to ascertain the law of the phenomena in relation to the

number of plates and the angle of incidence at which the polarisation was effected.

For this purpose, I provided myself with forty-seven plates of crown glass, each of which was about three inches long and one broad, and having formed them successively into parcels of 47, 44, 41, 39, &c. down to eight plates, I measured by means of a theodolite the different angles at which a pencil of light was polarised when transmitted through these various parcels. The results which were thus obtained, indicated at once a regular progression, and upon comparing the number of plates with the angles, it was obvious that the number of plates were always to one another, as the cotangents of the angles of incidence at which they polarised the transmitted light; that is, if n, n' represent the number of plates in any two parcels, and ϕ, ϕ' the angles at which the pencil was polarised, we have

$$n : n' = \text{cotang. } \phi : \text{cotang. } \phi' \text{ and}$$
$$n \times \text{tang. } \phi = n' \times \text{tang. } \phi'.$$

Hence it follows that the *number of plates in any parcel multiplied by the tangent of the angle, at which it polarises light, is a constant quantity*. From a great number of observations, made with a parcel of eighteen plates, I have found the constant quantity for crown glass to be 41.84, so that we have

$$\text{tang. } \phi = \frac{41.84}{n}$$

that is, divide the constant quantity by any given number of plates, and the quotient will be the natural tangent of the angle at which that number will polarise a pencil of light. In this way I have constructed the following table, shewing the various angles of polarisation from one plate up to

8,640,000 plates, the number by which light is polarised at an incidence of a single second.

The *first column* contains the number of plates in each parcel; the *second* the angles of incidence at which each parcel polarises the transmitted light, calculated from the preceding formula, and extending to parcels below eight, and above forty-seven, which could not easily be made the subject of experiment; the *third* column contains the experimental results from which the law was deduced; and the *fourth* exhibits the differences between the calculated and observed angles. The differences are all within the limits of error, and are singularly small when we consider the difficulty of observing the complete extinction of a luminous object, when the light by which it is formed has traversed a great number of plates. When the angle of incidence exceeds the angles of polarisation contained in the table, the pencil of light still emerges in a polarised state.

TABLE *shewing the Angles at which Light is polarised by oblique Refraction, through different numbers of Plates.*

Number of Plates in each Parcel.	Angles of Incidence at which Light is polarised. By Calculation.	Angles of Incidence at which Light is polarised. By Experiment.	Differences between the calculated and observed Angles.
1	88° 38′		
2	87 16		
3	85 54		
4	84 32		
6	81 50		
8	79 11	78° 52′ *	0 19′ —
10	76 33	76 24	0 9 —
12	74 0	74 2	0 2 +
14	71 30	72 15	0 45 +
16	69 4	69 40	0 36 +
18	66 48	66 48	0 0
21	63 21	63 39	0 18 +
24	60 8	61 0	0 52 +
27	57 10	56 58	0 12 —
29	55 16	54 50	0 26 —
31	53 28	53 16	0 12 —
33	51 44	51 0	0 44 —
35	50 5	50 23	0 18 +
39	47 1	46 50	0 11 —
41	45 35	45 49	0 14 +
44	43 34	44 0	0 26 +
47	41 41	42 0	0 19 +
100	22 42		
200	11 49		
500	4 47		
1000	2 24		
2000	1 12		
4000	0 36		
14000	0 1		
8,640,000	0 0 1″		

* This result was obtained by a parcel of plates of parallel glass.

By means of the preceding table, we may readily determine the quantity of light that is polarised at any angle φ, by a given number of plates n. Let m be the number of plates necessary to polarise the whole of the transmitted light, as found either from the table, or from the formula

$$m = \frac{41.84}{\text{tang. } \varphi}$$

L the quantity of light transmitted through the plates n, then

$$m : n = \text{L} : \frac{n\text{L}}{m}$$

the quantity of light required. If $\varphi = 43° 33'$; $\text{L} = 1$ $n = 2$ and $m = 44$, then we shall have $\frac{n\text{L}}{m} = 0.04545$ the quantity of light polarised by two plates at an angle of $43° 33'$.

The law of polarisation being thus ascertained, my next object was to determine the law of the variations which the angles experienced by changing the refractive power of the plates. From the difficulty of procuring ground plates of any substance of a high refractive power, I have made this experiment only in a rude manner with a parcel of twelve plates of green glass, which is nearly equal to flint glass in refractive power. The result of this experiment indicated that as the refractive power increased, the transmitted light was polarised at a less angle of incidence; but I cannot yet state the precise law till I have performed a series of experiments with a parcel of plates of flint glass which I am now preparing.

If a series of plates of glass is employed in the experiments on the elliptical coloured rings produced by topaz, either in polarising the incident rays in place of the reflecting plane, or in examining the rings in place of the agate and Iceland spar, the phenomena retain the same character; but are more distinct and

brilliant. The indistinctness occasioned by the interference of the two images in the case of Iceland spar, and of the bright and nebulous light in the case of the agate, is here in a great measure avoided.

The polarisation of light for this class of experiments may be effected in a very beautiful manner, by introducing into a glass trough about two-tenths of an inch wide, the fragments of a globe of glass blown to the utmost thinness, or the elementary films of which mica is composed. I have produced the same effect by coats of grease, by thick plates of mica,* by folds of gold beaters' skin,† and even by gold leaf itself. The gold beaters' skin is particularly fitted for this experiment when a strong light is used, as it disperses and equalises the light, and thus exhibits the rings to peculiar advantage.

Owing to the reflection which takes place at the surface of each plate, the image of a luminous object seen through a parcel of fifteen plates is encircled with a great number of faint images, which exhibit new phenomena. When the plates are placed at a little distance from each other in a wooden trough ABDC, Pl. VIII., fig. 1, a luminous object at O will be seen by the eye at E in the direction EE′, encircled with these faint images. By turning the trough round D, in the direc-

* The following measures were taken with two thick pieces of mica.

	Thickness.	Angle of Polarisation.
Mica - -	0.127 inch -	- 63° 0′
Ditto, another specimen	0.093 -	- 71 45

† The angle at which the complete polarisation of the transmitted light is effected may, in some instances, be employed as a good measure of the refractive power of the polarising plates or films. In the case of gold beaters' skin, and other substances which neither reflect nor refract light regularly, it is the only method which can be put in practice.

tion CAB, so as to increase the obliquity of the light radiating from O, the corner A will conceal the bright image at O, while the faint secondary images continue visible. If we now examine these images with a prism of Iceland spar, they will be found to be polarised exactly like the bright image; but upon increasing the obliquity of the incident rays still more, a *nebulous* light makes its appearance in the place of the faint images, and what is very singular, *this nebulous light is polarised in the opposite manner from the bright image, and the bright image has the same relation to this nebulous light, as the bright image has to the nebulous image of the agate, or as the first pencil has to the second pencil formed by doubly refracting crystals.*

When a beam of light from a candle is transmitted through a convex lens, and falls upon a series of plates at a considerable angle of incidence, a bright image A, and a row of faint images *cd* are distinctly visible, as in fig. 2. If we now view these images through a prism of Iceland spar, they will all be doubled; but upon turning round the prism, the image A will vanish, and along with it the faint images *ef*, and after another quarter of a revolution the image B will vanish, and along with it the faint images *cd*, the images A, and *ef* having reappeared. When the angle of incidence, however, is greatly increased, the secondary images *cd* always vanish along with their principal image A, and *ef* along with their principal image B.

The polarisation of the nebulous light, in an opposite manner from the bright image, first shewed itself to me in a very curious phenomenon. When the nebulous light and the bright image were both visible at the same time, and were transmitted through topaz, so as to form the elliptical coloured

rings, I was surprised to observe the lower half of all the rings completely different from the upper half, like the sketch in fig. 3, the half of the rings above the conjugate diameter of each was the very same as the *first set* described in my former paper, while the other half below the conjugate diameter contained the complementary rings, such as those which form the *second set* in the same paper.

When we polarise a beam of light by refraction, at the same angle at which it is polarised by reflection, we obtain very interesting results. The number of plates necessary for this purpose is thirty, as represented by ABCD, fig. 4. If a ray of light RS is incident at S, so that the angle of incidence is 54° 35′, then, according to the observations of MALUS, the reflected ray ST will be wholly polarised, while according to the preceding experiments, the transmitted beam EF will also be completely polarised. The pencil EF, however, is polarised in the opposite manner to ST, and the two pencils have the same relation to each other, as the two images formed by double refraction. If we, therefore, suppose the thickness AC of the thirty plates diminished to such a degree, that the eye could receive both the pencils ST and EF, and if we examine these two images by a prism of Iceland spar, they will comport themselves in every respect, as if they had been produced by a doubly refracting crystal, vanishing and reappearing alternately in every quadrant of the circular motion of the spar.

As a portion of the transmitted beam SEF is polarised at its emergence from each plate, and as this portion possesses a polarisation opposite to that of the reflected beam ST, and falls upon the subjacent plates at an angle of 54° 35′, *not one particle of it will suffer reflection, but each minute portion of light*

polarised by every successive plate will force its way through all the remaining plates, and will reach the eye at E, *without having lost a single ray by reflection, the light reflected at each surface being taken from the unpolarised portion of the transmitted light.* Hence it follows, that the light which penetrates through a parcel of plates is not a maximum when it falls with a perpendicular incidence; that the principle employed by BOUGUER for computing the intensity of the light transmitted by several plates is completely erroneous; and that the method adopted by the same distinguished philosopher for measuring the absorption of light is affected with a similar error.*

The celebrated discovery made by MALUS, of the polarisation of light by oblique reflection, is perhaps the most important that optics has received since the discovery of the principle of the achromatic telescope; but though it developed a new set of phenomena, analogous to those produced by doubly refracting crystals, yet as the polarisation of one of the images, at least, formed by these crystals, was effected by *refraction*, and not by *reflection*, it did not furnish us with any information respecting the manner in which they polarised the transmitted light. The discovery, however, of the polarisation of light by oblique refraction, forms the connecting link between these two classes of facts, and holds out the prospect of obtaining a direct explanation of the leading phenomena of double refraction.

* In estimating the quantity of light absorbed, BOUGUER (*Traité d'Optique sur la gradation de la Lumiere*, p. 156—160,) compared the quantity transmitted by four pieces of glass with the quantity transmitted by one piece having the same thickness with all the four. The pencil of light was incident at an angle of 15°, and hence the 177th part of the light transmitted by each plate was polarised, and therefore not altogether subject to the general law of reflection.

If two separate sets of polarising plates are placed before two luminous objects, in such a manner that their planes of refraction are perpendicular to each other, and that the rays are incident upon both parcels at the polarising angle belonging to each, the observer will perceive two images which have all the properties of those formed by doubly refracting crystals; and in so far as the polarisation of the two pencils is concerned, the two parcels of plates form an *artificial polarising crystal*. For if these two images are viewed through a prism of Iceland spar, they will be converted into four images, and by turning round either the spar, or the plates, two of the images will vanish and reappear in every quarter of a revolution.

From these observations, we are entitled to conclude, that the mystery of the polarisation of the two pencils formed by double refraction is completely removed by a reference to a more general principle, and that the formation of two images and the oblique refraction at a perpendicular incidence are the only parts of the problem which now require a solution.

From the view which has just been given of the nature and consequences of the polarisation of light by oblique refraction, you will readily see that the prosecution of these experiments will lead to results of still higher interest. The establishment of a new law must at all times be considered as an important step in the progress of science, but when this law presides over a class of facts, all of which are unexplained, and many of which still remain to be discovered, it claims a higher regard both as an instrument of discovery, and as a principle for explaining new and analogous phenomena.

If this paper shall meet with your approbation, and that of

the Royal Society, I shall communicate to you the results of a set of experiments on the *Polarisation of Light by reflection*. In this paper I shall be able to shew, by the most satisfactory details, that the principle discovered by MALUS is not, as has been supposed, a general law, but that it depends upon a circumstance which he appears to have completely overlooked, viz. the relation between the quantities of reflected and transmitted light when the pencil is incident at the polarising angle. When light is polarised by reflection from water, the reflected light is only $\frac{41}{1000}$ part of the incident pencil, and in the case of glass it is only $\frac{11}{1000}$, but when *realgar*, *diamond*, and *chromate of lead* are employed, the light reflected at the polarising angle is above $\frac{500}{1000}$, and therefore none of these bodies have the power of polarising the whole of the reflected pencil.

I have the honour to be,

Sir, your most obedient humble servant,

DAVID BREWSTER.

Edinburgh, Nov. 24, 1813.

To TAYLOR COMBE, Esq. Sec. R. S.

Fig. 1.

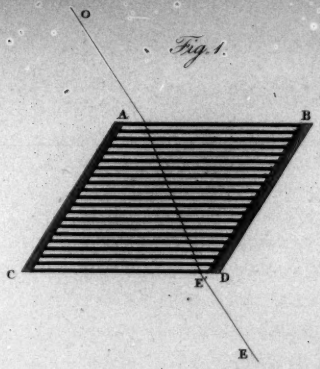


Fig. 2.

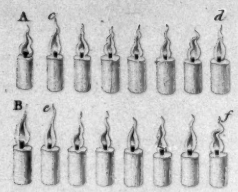


Fig. 3.

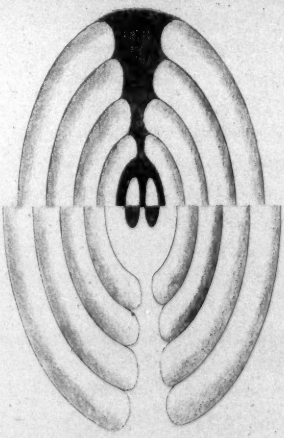
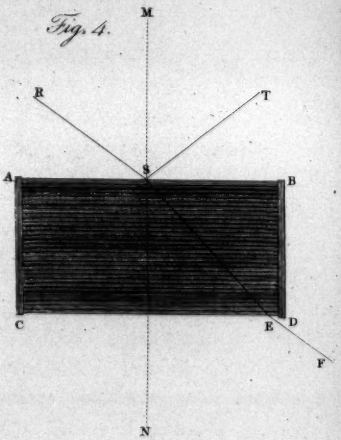


Fig. 4.



XIII. *Further Experiments on the Light of the Cassegrainian Telescope compared with that of the Gregorian. By Captain* Henry Kater, *Brigade Major. In a Letter addressed to the Right Hon. Sir* Joseph Banks, *Bart. K. B. P. R. S.*

Read November 18, 1813.

Sir,

Since you did me the honour of submitting to the Royal Society my experiments on the comparative light of the Cassegrainian and Gregorian telescopes, I have been so fortunate as to have an opportunity of establishing the conclusions deduced from them by another experiment, the detail of which I trust will not prove unacceptable.

The telescopes used on the present occasion were the excellent Gregorian described in my former paper, and a new Cassegrainian constructed by Mr. Crickmore. The diameter of the large speculum of this instrument is 4.9 inches, the distance between the mirrors only 18 inches, and it bears a power of about 550 (as roughly determined by experiment) with perfect distinctness and considerable light.

The magnifying powers used were 157 in the Cassegrainian, and 125 in the Gregorian; these were carefully determined by experiment. A circular piece of pasteboard was then prepared to close the end of the Cassegrainian telescope, and concentric circles were drawn on it at the distance of the twentieth of an inch from each other. These were succes-

sively cut out till a card appeared decidedly brighter when viewed through the Cassegrainian than through the Gregorian.

The following measures were then taken.

Cassegrainian Telescope.	Inch.	Gregorian Telescope.	Inch.
Diameter of the circular opening in the paste-board - -	3,600	Diameter of the large mirror -	3,950
Diameter of the back of the small mirror	1,900	Diameter of the back of the small mirror	1,230
Length of the arm	0,850	Length of the arm	1,155
Thickness of ditto	0,225	Thickness of ditto	0,175
		Length of a bar containing the adjustment	0,700
		Its width -	0,150
		Diameter of three semicircles used as rests for the great mirror	0,375

From the above data the following calculations were made.

	Inch.		Inch.
Area of the circular opening in the paste-board -	10,179	Area of the large mirror - -	12,254
Area of the back of the small mirror to be deducted -	2,896	Area of the back of the small mirror to be deducted -	1,188
Area of the arm to be deducted	0,191	Area of the arm to be deducted	0,202
	3,027	Area of the bar containing the adjustment to be deducted	0,105
		Area of the three semicircles to be deducted	0,166
			1,661
Area of the portion of the mirror exposed to the light -	7,152	Area of the portion of the mirror exposed to the light -	10,593

The magnifying powers being 157 in the Cassegrainian telescope and 125 in the Gregorian, their comparative light when reduced to the same aperture and power will be as $\frac{10.593}{125^2}$ to $\frac{7.152}{157^2}$, or as 678 to 290.

It may perhaps be desirable to place the results of the three experiments in one point of view. Thus, calling the light of the Gregorian telescope in every instance 100, we have the light of a telescope of the Cassegrainian form of equal power and aperture, by the first experiment 235, by the second 148, and by the third 234.

The surprising agreement of the first and third experiments with each other, excites an inquiry as to the cause of the difference observable in the second, as equal care and attention were bestowed on all.

The Cassegrainian telescope used on that occasion I have been unable to procure again, but from the inquiries I have since made, there is some reason to believe that, in addition to the polish of the specula having been somewhat impaired, as remarked in my last paper, its convex mirror was too small to receive the whole cone of rays, and to this circumstance principally I am inclined to attribute the deficiency of light in the second experiment when compared with the first and third.

If the mean of all three experiments be taken, the relative quantity of light will be as $20\frac{1}{2}$ to 10; but, if the second experiment be rejected, and the mean of the first and third be considered as correct, the light will be very nearly as $23\frac{1}{2}$ to 10 in favour of a telescope of the Cassegrainian form compared with one of the Gregorian construction.

I have the honour to remain, &c. &c.

Ipswich, 16th July, 1813. HENRY KATER.

P. S. A fact which appears to be somewhat analogous to that which I have observed, has been remarked by Dr. BREWSTER in his late treatise on Philosophical Instruments. "It is a curious fact," he says, p. 44, "that the circular images or the sections of the cone of rays, are never so distinct and well defined after the rays have crossed—as they are before—;" and again, p. 193, "the sections of the cones of light are, in general, better defined, when they are taken between the object glass and its principal focus."

APPENDIX.

SINCE the preceding paper was submitted to the Royal Society, I have made numerous experiments connected with the same subject for the purpose of bringing the results I then deduced to the test of a further examination, conducted in a different manner, and with a more simple apparatus.

My object was to determine the intensity of light reflected from a concave mirror at equal distances from its focus, and thus to decide, in a direct manner, whether any rays are lost in crossing each other at the place where an image is formed.

The mirror employed in these experiments belongs to an excellent Newtonian telescope; its diameter is 4,6, and its solar focus 29,5 inches. This mirror was supported by means of a cylindrical rod of iron screwed to the back of the cell which contains it, on two upright pieces of wood mortised into a square board, which thus formed a frame in which the mirror might be fixed in any required position.

The mirror being placed to receive the rays of the sun, I held a card at such a distance within the focus, that the luminous circle formed by reflection was equal in diameter to the width of the card; and having endeavoured to fix in my mind the degree of illumination, I suddenly removed the card far beyond the focus, and then brought it towards the mirror till the diameter of the luminous circle was the same as before. In this position the degree of illumination appeared so far inferior to that before observed, as to admit of no hesitation in the decision.

Before I proceed to detail the experiments which follow, I must premise that of the eight persons who at different times were employed in making the observations, six were perfectly unacquainted with the subject of inquiry, and two of the six distinguished in the column of observers by the letters C, and W, were servants. It will be seen, that I did not trust myself to make any observations till the fact I sought had been sufficiently established by others to leave no fear on my mind that I might be influenced by any preconceived opinion.

I drew a line across a card, which line I divided into tenths of an inch. The mirror was placed at the distance of fourteen feet from a candle, and the card moved in the reflected light *within* the focus till the diameter of the luminous circle was equal to the width of the card, in which position an observer was directed to remark carefully the intensity of the illumination. The card was then suddenly carried far *beyond* the focus and brought slowly towards it till the observer pronounced the intensity of the light to be the same as before, at which instant I noted the diameter of the luminous circle by means of the divided line drawn on the card: the square of this

diameter compared with the square of the diameter of the first circle, gave the ratio of the intensity of light at equal distances from the focus.

Observers.	Diameter of the illuminated circle *within* the focus.	Diameter of the circle of equal illumination *without* the focus.	Ratio of the intensity of the light at equal distances from the focus.	
			Within.	Without.
A	24	17	1000	502
B	25	17	1000	445
K	25	17	1000	445
C	25	18	1000	518
		Mean	1000	477.5

I now caused a joint to be made in front of the board supporting the mirror, which received the end of a light rod of wood about three feet in length. On this a thin board was contrived to slide easily at right angles, and the rod moving stiffly in the joint, could be set to any elevation so as to receive the light reflected from the mirror on the thin board. The board was covered with white paper divided by perpendicular lines at the distance of the tenth of an inch from each other. The observations were then made, and the ratio of the light deduced in the same manner as in the preceding experiments.

Observers.	Diameter of the illuminated circle *within* the focus.	Diameter of the circle of equal illumination *without* the focus.	Ratio of the intensity of the light at equal distances from the focus.	
			Within.	Without.
Y	24,3	15,75	1000	420
Y	24,3	16,00	1000	433
K	24,3	14,50	1000	352
K	24,3	16,25	1000	447
K	24,3	16,25	1000	447
M	24,3	16,00	1000	433
M	24,3	16,50	1000	461
M	24,3	15,00	1000	381
		Mean	1000	421,6

Not being perfectly satisfied with the manner in which these experiments were conducted, I was anxious to devise some means of viewing the light on both sides of the focus at the same time; and this to a certain degree I accomplished in the following manner.

I caused two small blocks of wood to be made to slide easily on the rod beforementioned, and cut in each a notch with a fine saw to receive a slip of card at right angles to the rod. On each card a circle of half an inch in diameter was drawn which remained white, the rest of the card being painted black. By this contrivance, I could receive on each circle a portion of the light reflected from the mirror when the slips of card were placed one within the focus, and the other without. The one without the focus remained stationary, whilst that within was shifted till the observer pronounced the white circles to be equally illuminated, when the distance between the cards was

carefully measured. At the conclusion of these observations, the card next the mirror was moved till a perfect image of the candle was formed on it, when its distance from the card beyond the focus was measured, and the mean of a number of these last observations (differing but little from each other) was taken, in order to ensure a more accurate result. The distance of each card from the focus when the light was of equal intensity being thus obtained, the squares of those distances will express the relative intensity of illumination at equal distances on either side of the focus.

With white circles on black cards.

Observers.	Distance of the white circles when equally illuminated.	Distance of the white circle from the focus *without*.	Distance of the white circle from the focus *within*.	Ratio of the intensity of the light at equal distances from the focus.	
				Within.	Without.
Myself	25,25	10,	15,25	1000	480
——	25,75	10,	15,75	1000	403
——	25,80	10,	15,80	1000	427
K	25,13	10,	15,13	1000	437
——	26,25	10,	16,25	1000	379
——	25,25	10,	15,25	1000	480
C	24,25	10,	14,25	1000	492
——	25,37	10,	15,37	1000	423
——	24,75	10,	14,75	1000	460
			Mean	1000	481,2

Conceiving that some error might possibly arise from the one circle being nearer to the eye than the other, I substituted for them transparent disks of six-tenths of an inch diameter,

made by cutting circular holes in two slips of black card paper, and covering them with fine oiled paper. Small shades of black card paper were contrived to shield the disks from the direct rays of the candle. The disk within the focus being made the brightest, was moved at intervals towards the mirror till the observer pronounced both disks to be equally illuminated, when their distance was registered as the "*first reading*." The disk within the focus was then advanced so near the mirror, as to appear much the faintest, and afterwards brought back gradually till the illumination of the disks was a second time pronounced to be equal, when their distance was again measured and entered under the head of "*second reading*," and the same method was followed in all the subsequent experiments.

With large transparent disks.
Distance of the candle from the mirror 186 inches.

Observers.	Distance of the disks when equally illuminated.		Distance of the disk from the focus *without*.	Distance of the disk from the focus *within*.	Ratio of the intensity of the light at equal distances from the focus.	
	1st reading.	2d reading.			Within.	Without.
Myself	26,45	25,70	10, 8	15,77	1000	427
K	24, 5	2 5, 6	10,22	14,83	1000	475
				Mean	1000	451

In making the third set of experiments I remarked that a part of the light from the mirror was received on that portion of the card which was blackened, and fearing that this circum-

stance by diverting the attention from the white circles might occasion an inaccurate decision, I prepared two white disks of card paper half an inch in diameter, which were supported on black wires, and these I found far preferable to every other contrivance which I employed. Care was taken to place the disks as near the axis of the mirror as was possible without their interfering with each other.

The following were the results.

With disks of card paper supported on wires.

Distance of the candle from the mirror 186 inches.

Observers	Distance of the disks when equally illuminated.			Distance of the disk from the focus *without*.	Distance of the disk from the focus *within*.	Ratio of the intensity of the light at equal distances from the focus.	
	1st reading.	2d reading.	Mean.			Within.	Without.
W	25,07	24,95	25,01	9,8	15,21	1000	415
K	23,90	24,85	24,37	9,8	14,57	1000	452
Myself	24,80	25,00	24,90	9,8	15,10	1000	421
K	23,40	24,95	24,17	9,8	14,37	1000	465
W	23,70	24,35	24,02	9,8	14,22	1000	475
Myself	23,82	24,60	24,21	9,8	14,41	1000	463
					Mean	1000	448,5

Imagining that the superior whiteness of plaister of Paris might render a slight difference of light more perceptible than it could be by the card disks, I prepared two balls of this substance by casting them in a bullet mould, and supported them on black wires. With these the following experiments were made.

With Plaister of Paris balls.

Distance of the candle from the mirror 186 inches.

Observers	Distance of the balls when equally illuminated.			Distance of the balls from the focus *without*.	Distance of the balls from the focus *within*.	Ratio of the intensity of the light at equal distances from the focus.	
	1st reading.	2d reading.	mean.			Within.	Without.
W	24,20	24,48	24,31	10,	14,31	1000	488
K	25,50	24,8	25,15	10,	15,15	1000	436
Myself	25,40	25,67	25,53	10,	15,53	1000	415
W	24,77	25,60	25,18	10,	15,18	1000	434
K	24,77	24,15	24,46	10,	14,46	1000	478
Myself	24,85	24,77	24,81	10,	14,81	1000	456
					Mean	1000	451,1

Lastly, having made only two experiments with transparent disks, and those being of an inconvenient size, I prepared others four-tenths of an inch in diameter, which were constructed and used as before.

With small Transparent Disks.

Distance of the candle from the mirror 186 inches.

Observers	Distance of the disks when equally illuminated.			Distance of the disk from the focus *without*.	Distance of the disk from the focus *within*.	Ratio of the intensity of the light at equal distances from the focus.	
	1st reading.	2d reading.	mean.			Within.	Without.
W	23,85	26,50	25,17	10,26	14,91	1000	474
K	25,20	25,65	25,42	10,26	15,16	1000	458
Myself	25,37	25,43	25,40	10,26	15,14	1000	459
					Mean	1000	463,7

If the mean results of the various experiments be now collected they will appear as follows:

1st set.	By the diameters of the circles of illumination - - -	1000	477,5
2d —	By ditto - - - -	1000	421,6
3d —	By white circles on black cards	1000	431,2
4th —	By large transparent disks -	1000	451,0
5th —	By disks of card paper on wires	1000	448,5
6th —	By Plaister of Paris balls -	1000	451,1
7th —	By small transparent disks -	1000	463,7
	Mean	1000	449,2

The following method, by which both disks may be seen at the same instant, was suggested by a friend.

Two lamps were procured, and the wicks being trimmed so that the flames appeared of an equal size, one lamp was placed 16 feet from the mirror, and the other 4 feet, which

last distance, as will be seen, was varied in the course of the experiments. Two small disks of card paper a quarter of an inch in diameter, supported on wires, were fixed on one of the sliders, one inch and three quarters from each other, and both at the same distance from the mirror. The slider being placed between the two foci of the lamps, they were arranged so that the reflected light from the most distant lamp fell on one of the disks *after* the rays had crossed at the focus, and that from the nearest lamp was received by the other disk *before* the rays arrived at the focus. The slider was then moved between the foci till the disks appeared equally illuminated, when the division (the rod being divided into inches and tenths) cut by the slider was registered. The slider was then moved till a well defined image of the furthest lamp appeared on the disk, and the division cut by the slider being noted, the distance of the disks, when equally illuminated, from the focus of the furthest lamp was readily obtained. The foci of the two lamps being calculated, their difference was known, and by means of it, and the distance of the disks from the focus of the furthest lamp, their distance from the focus of the *nearest* lamp was determined. The mean of several readings was in every instance taken to insure a more accurate result, and the lamps were changed during each experiment, in order to obviate any error which might arise from the inequality of their light. The rays from the distant lamp were first thrown on the right hand disk, and in the subsequent series on the left; by which the error arising from any shade of difference in the disks (if such existed) was corrected.

Let L, be the distance of the furthest lamp.

l, That of the nearest.

f, The focus of the furthest lamp.

F, The focus of the nearest lamp.

d, The distance of the disks from the focus of the *furthest* lamp when equally illuminated.

D, The distance of the disks from the focus of the *nearest* lamp when equally illuminated.

Then the relative intensity of the light at *equal* distances on each side of the focus will be $\left(\frac{f}{L \times d}\right)^2 : \left(\frac{F}{I \times D}\right)^2$.

With disks of card paper. Distance of the furthest lamp 16 feet.

Distance of the nearest lamp.	Rays from the furthest lamp on the right hand disk.			Rays from the furthest lamp on the left hand disk.		
	Distance of disks from the focus when equally illuminated.		Intensity of light without the focus, that within being 1000.	Distance of disks from the focus when equally illuminated.		Intensity of light without the focus, that within being 1000.
	Without.	Within.		Without.	Within.	
5	1,74	8,77	804	1,75	8,76	815
6	1,47	5,76	763	1,37	5,66	901
7	1,14	4,10	587	1,32	3,92	857

The mean of these experiments gives the intensity of light within the focus to that without, as 1000 : 788.

Disks of glass ground on both sides were now substituted for those of card paper, and with these the following experiments were made with the utmost care.

With disks of ground glass. Distance of the furthest lamp 16 feet.

Rays from the furthest lamp on the right hand disk.				Rays from the furthest lamp on the left hand disk.		
Distance of the nearest lamp.	Distance of disks from the focus when equally illuminated.		Intensity of light *without* the focus, that within being 1000.	Distance of disks from the focus when equally illuminated.		Intensity of light without the focus, that within being 1000.
	Without.	Within.		Without.	Within.	
4	2,58	14,28	1441	1,71	15,15	563
5	1,80	8,71	872	1,52	8,99	584
6	1,36	5,87	629	1,34	5,89	606
7	1,16	4,08	614	1,17	4,07	627

From these experiments, the intensity of light within the focus to that without appears to be, as 1000 : 742; but as the first experiment is evidently erroneous, if it be rejected, the intensity of the light within the focus to that without, will be as 1000 : 655.

The difference of these results from those obtained by the methods detailed in the former part of this paper, led me to examine the intensity of the light of both cones of rays *within* the foci. For this purpose, the following experiments were made.

With disks of ground glass *within the foci*.
Distance of the furthest lamp 16 feet.

Distance of the nearest lamp.	Rays from the furthest lamp on the right hand disk.			Rays from the furthest lamp on the left hand disk.			
	Distance of the disks when equally illuminated from the focus of the furthest lamp.	Distance of the disks when equally illuminated from the focus of the nearest lamp.	Intensity of the light within the focus of the furthest lamp, that within the focus of the nearest being 1000.	Distance of the disks when equally illuminated from the focus of the furthest lamp.	Distance of the disks when equally illuminated from the focus of the nearest lamp.	Intensity of the light within the focus of the furthest lamp, that within the focus of the nearest being 1000.	Mean.
4	3,38	20,24	1231	3,14	20,00	1089	1160
5	3,22	13,73	1124	3,23	13,74	1129	1126
6	3,30	10,53	1151	3,16	10,39	1084	1117
7	3,26	8,50	1117	3,12	8,36	1058	1087
8	3,19	6,99	1108	2,98	6,88	998	1053

From these last experiments, it should seem that a portion of the light forming the cone of rays from the nearest lamp is destroyed by the interference of the cone of rays from the furthest lamp; but this is an inference too improbable to be received as conclusive without further experiments.

The preceding method of determining the proportional intensity of light without the focus to that within, is not much to be relied upon, from the circumstance of the light after it has crossed at the focus appearing of a *deeper yellow colour* than before, which renders it extremely difficult to decide when the disks are equally illuminated; the circle of light, too, from the furthest lamp being very small so near the focus, and its illumination unequal in different parts, forms another source

of error; from these objections, however, I must observe, the experiments made within both foci are in a great measure exempt.

With respect to the experiments detailed in the preceding part of this paper, it may be remarked in their favour, that the observations were made by eight different persons at various times, and by different methods, and that the near coincidence of the results with each other, and with the numbers deduced from the experiments made with the Cassegrainian and Gregorian telescopes, seem to warrant the conclusion that a mean of the whole may be considered as a tolerable approximation to the truth.

London, April 20, 1814.

XIV. *Astronomical Observations relating to the sidereal part of the Heavens, and its Connection with the nebulous part; arranged for the purpose of a critical Examination.* By William Herschel, *LL.D. F.R.S.*

Read February 24, 1814.

In my paper of observations of the nebulous part of the heavens, I have endeavoured to shew the probability of a very gradual conversion of the nebulous matter into the sidereal appearance The observations contained in this paper are intended to display the sidereal part of the heavens, and also to shew the intimate connection between the two opposite extremes, one of which is the immensity of the widely diffused and seemingly chaotic nebulous matter; and the other, the highly complicated and most artificially constructed globular clusters of compressed stars.

The proof of an intimate connection between these extremes will greatly support the probability of the conversion of the one into the other; and in order to make this connection gradually visible, I have arranged my observations into a series of collections, such as I suppose will best answer the end of a critical examination.

I. *Of Stars in remarkable situations with regard to Nebulæ.**

Among the great number of stars, with nebulosity dispersed between them are some in situations that deserve to be remarked.

IV, 5 is "A pretty bright star situated exactly north of the "center of an extended milky ray, which is about 15 or 20 "minutes in length." By a second observation, two years after the first, it appeared that the star was then included in part of the nebulosity.

V, 46 is "A pretty bright star in the middle of a very bright "nebula, about 10 minutes in length and 2′ broad." See fig. 1.

III, 616 is "A star of the 6th magnitude, about 5′ north of "a very faint nebula, of an irregular figure." By an observation of the same star, two years before, the two objects were then so near each other, as, at first sight, to cause a suspicion that some damp had settled upon the eye-glass and affected the star.

The singularity that five stars should be similarly situated with regard to nebulæ is not very striking; but the difference in the additional observation is worthy of notice, and may suggest a surmise that nebulæ may have considerable proper motions, by which they are occasionally carried towards neighbouring stars: the difference in the clearness of the atmosphere

* See five stars in remarkable situations, II, 246. III, 201. 616. IV. 5. 46. The places of these objects will be found in three catalogues of nebulæ and clusters of stars, published in the Philosophical Transactions for 1786, 1789, and 1802. The reference IV, 5 for instance, points out number 5, in the IVth class.

at different times, however, ought to make us cautious about assigning the cause of the difference in the observations.

2. *Of two Stars with nebulosity between them.**

A more remarkable situation than the former is that of two stars with nebulosity between them, or both included in the same nebulosity.

III, 67 is "An extremely faint nebulosity extended from "one star to a smaller one, at the distance of about 2 minutes "south of the former." See fig. 2.

II, 706. "Two considerable stars are involved in a very "faint nebulosity of 3 or 4 minutes in extent." See fig. 3.

Here I have referred to 19 instances, where two stars have an extended nebulosity between them, or at least are both contained within it. Now, if we were to enter into a calculation of chances to investigate the probability that in every one of these 19 objects, the stars and the nebulosity should be unconnected, we should have to consider that in order to produce this appearance by three objects at a distance from each other, it would be required that every one of them should be precisely in a given line of sight, and that the nebulosity should not only be in the middle of them, but that it also should be extended from the situation of one star to that of the other; and that all this should happen in the confined space of a few minutes of a degree; which cannot be probable. Then, if on the other hand we recollect that in the 8th, 9th, and 10th articles of my paper on the nebulous part of the heavens, I have given 139 double nebulæ joined by nebulosity between them,

* See nineteen double stars joined by intermediate nebulosity II, 16. 706. 732. III, 19. 32. 67. 68. 113. 126. 182. 200. 312. 376. 540. 637. 757. 785. 820. 854.

and that we have now before us 19 similar objects, with no other difference than that instead of nebulæ we have stars with nebulosity remaining between them, should we not surmise that possibly these stars had formerly been highly condensed nebulæ, like those that have been mentioned, and were now by gradually increasing condensation turned into small stars; and may not the nebulosity still remaining shew their nebulous origin?

When to this is added that we also have an account of 700 double stars entirely free from nebulosity,* many of which are probably at no great real distance from each other, it seems as if we had these double objects in three different successive conditions: first as nebulæ; next as stars with remaining nebulosity; and lastly as stars completely free from nebulous appearance.

8. *Of Stars with nebulosities of various shapes attached to them.*

When a nebula seems to be joined to a star, or closely pointing to it, the manner of its appearance deserves our attention. Here follow three different sorts of such conjunctions.†

First sort; I, 148 "On the north preceding side of a pretty "bright star is a considerable, bright nebulosity. It is joined "to the star so as to appear like a brush to it. See fig. 4."

Second sort; IV, 4 "A very small star has an extremely

* See Phil. Trans. for 1782, page 12; and for 1785, page 40.

† See fourteen stars connected with nebulæ.
Nine with a brush I, 143. II, 214. 683. III, 643. IV, 10. 17. 29. 40. 77.
Two with a puff IV, 4. 3.
Three with fan-shaped nebulosity IV, 2. 35. 66.

"faint, and very small nebula attached to it in the shape of "a puff." See fig. 5.

Third sort; IV, 35 "A small star has a small, faint, fan-"shaped nebulosity joining to it on the north preceding side." See fig. 6.

Here we have a list of fourteen objects, in which the probability of a union between the nebulosities and the stars will gradually become more apparent. With regard to the first nine, the particularity of their construction is already very pointed: the conditions are that the nebulosity must be extended; the direction of its extension must be exactly towards the star, and it must also be apparently just near enough to touch it; but that all this should happen cannot be probable; whereas a real contact of the objects, held together by mutual gravitation, will readily account for the whole appearance.

In the two next objects there is already some indication of a union between the nebula and the star, for the roundness of the nebulosity appeared to be a little drawn out of its figure towards the star.

But the last three instances, in which the whole mass of nebulous matter is pointedly directed to the stars, and in contact with them, can hardly leave any room for doubting a union between them.

Now if we admit a contact, or union between these nebulæ and the stars, it deserves to be remarked that stars, in the situation of these fourteen, cannot have been formed from their adjoining nebulosities; for a gradual condensation of the nebulous matter would have been central; whereas the stars are at the extremity of the nebulæ. It is therefore reasonable to suppose that their conjunction must be owing to some

motion either of the stars or of the nebulous matter: a mutual attraction might draw them together. In either of these cases it would follow, that if the nebulosity should subside into the star, as seems to be indicated by the assumed form of the fan-shaped nebulæ, the star would receive an increase of matter proportional to the magnitude and density of the nebulosity in contact with it. This would give us the idea of what might be called the *growth* of stars.

4. *Of Stars with nebulous branches.*

That an intimate connection between the nebulous matter and a star is not incompatible with their nature will clearly appear by the following instances, in which a union is manifested that cannot be mistaken for a deceptive appearance.*

IV, 42 is "A star of about the 8th or 9th magnitude with "very faint nebulous branches extended in the direction of the "meridian: each branch is about one minute in length. Other "stars of the same size, and at the same time in view, are "free from these branches." See fig. 7.

The three objects to which I have referred shew sufficiently that stars and nebulæ may be connected; for a little swelling and increase of light of the branches, at their junction with the star, which generally takes place, seems evidently to be an effect arising from the gravitation of the nebulous matter towards a center, in which the star is situated.

Here again the visible effect of gravitation supports the idea of the *growth* of stars by the gradual access of nebulous matter; for in the present case I may refer to the observations already published in the Phil. Trans. for 1811, where, page

* See three stars with nebulous branches IV, 42. 43. 48.

301, we have an account of twenty-four extended nebulæ, gradually a little brighter in the middle; page 303 there are fifty extended nebulæ, with an increased brightness towards the middle; page 304, we have fifty-four extended nebulæ, with a much greater accumulation of brightness; page 307 there are seven extended nebulæ, in which the central increase of brightness approaches towards the formation of a nucleus. Page 309, we have twenty-seven extended nebulæ, in which the central nucleus is already formed; and finally, page 311 contains the account of twenty-three extended nebulæ, where the nebulosity seems to have so far subsided into the nucleus, as to leave only two opposite faint branches. Who then that has followed up the gradual condensation of an extended nebula till it appeared in the shape of a bright nucleus with faint branches, and finds now in the center of two such opposite faint branches, instead of a condensed nucleus, a star—who, I may ask, would not rather admit that the nucleus had gradually cleared up in brightness, and assumed the lustre of a star, than have recourse to the most improbable of all hypotheses, that a fortuitous central meeting of a star and a nebula should be the cause of such a singular appearance?

5. *Of nebulous Stars.**

The conjunction of the nebulous and sidereal condition is still more clearly manifested in nebulous stars. Having already described many of them in a paper read before the Royal Society in 1791, I shall here only mention two of them.

* See thirteen nebulous stars IV, 19. 25. 36. 38. 44. 45. 52. 57. 58. 65. 69. 71. 74.

IV, 45. "A star of about the 9th magnitude has a pretty "strong milky nebulosity equally dispersed all around it."

IV, 69 is "A star of about the 8th magnitude with a faint "luminous atmosphere of a circular form of about 3 minutes "in diameter; the star is perfectly in the center, and the at- "mosphere is so diluted, faint, and equal throughout, that "there can be no surmise of its consisting of stars, nor can "there be a doubt of the evident connection between the "atmosphere and the star." See fig. 8.

Among the thirteen objects referred to, there are many so variously constructed as to prove not only that nebulous stars are intimately connected with a nebulosity, which from its great regularity might be taken for an atmosphere, but also with the luminous appearances, which have been described as belonging to the nebulous matter that is so widely expanded over various regions of the starry heavens. For instance, in IV, 45. 58. 65 and 69 the stars are perfectly central, which proves that the chevelure is connected with them. In IV, 86. 71 and 74 the nebulosity is likewise attached to the stars, but their nebulosity is more irregularly and extensively expanded, so as to resemble the general mass of nebulous matter.

What has been said of the gradual condensation of the nebulous matter in the case of extended nebulæ, is supported by a much greater number of nebulosities of a spherical form. The different gradations of their condensation are pointed out in the same paper from page 301 to 308; and contain 322 cases in which the fact of the gradual condensation is rendered so evident as not to admit of a doubt. Then, if instead of the last 13 globular nebulæ, page 309, each of which has a nucleus in the middle, we now look at our 13 stars, each of

which is situated in the very center of a globular nebulosity, it will evidently point out the high probability, or rather the certainty, that nebulous stars only differ from round nebulæ containing a nucleus, in the order of condensation, which in the case of the nebulous stars, has been carried to a somewhat higher degree than in the nebulous nuclei.

*6. Of Stars connected with extensive windings of nebulosity.**

The nebulosity which has been shewn to be connected with stars may be fully proved to be of the same nature as the general mass of nebulous matter by the following instances.

IV, 33. "A star situated upon a ground of extremely faint "milky nebulosity diffused over this part of the heavens, has "a milky chevelure surrounding it, which is brighter than "the nebulosity of the ground; but which loses itself imper- "ceptibly in the extreme faintness of the general diffusion of "the nebulous matter." See fig. 9.

The formation of these objects is extremely instructive, as it manifests the affinity between the matter of which stars are composed, and that of the most unshapen chaotic mass of nebulosity. For the vanishing chevelure of a star being equally connected, on the one hand, with the generally diffused nebulous matter, and on the other with the star itself, round which it is in a state of gradual condensation; this double union denotes the mutual gravitation of the whole mass of nebulosity and the star towards each other; and unless this proof can be invalidated, we must admit the fact of the growing condition of stars, that are in the situation which has been pointed out.

This argument also adds greatly to the probability of stars

* See three stars connected with diffusions of nebulosity IV, 24. 33. V. 27.

being originally formed by a condensation of the nebulous matter; for, as it now appears that stars must receive an addition to their solid contents, when they are in contact with nebulosity, there is an evident possibility of their being originally formed of it. Moreover, the affinity between the nebulous and sidereal condition being established by these observations, we may be permitted to conceive both the generation and growth of stars, as the legitimate effects of the law of gravitation, to which the nebulous matter, as proved by observation, is subject.

7. *Of small patches consisting of Stars mixed with nebulosity.*

When a small patch of stars is mixed with nebulosity, there is a possibility of its being a deception arising from their being accidentally in the same line of sight; but it has already been shewn that in such appearances the probability is much in favour of a real union; especially when the objects are numerous;* and in that case there are but two ways of accounting for it.

First, admitting from what has been said, that stars may be formed of nebulous matter, it may happen that the nebulosity still mixed with them is some remaining unsubsided part of that from which they were formed; and in the next place, a union of stars and nebulosity, originally at a distance from each other, may have been effected by the motion of either the stars or the nebulosity.

* See thirty-seven small patches, consisting of stars mixed with nebulosity I. 172. 192. 258. II. 21. 39. 103. 304. 489. 745. 878. III. 8. 43. 61. 64. 71. 143. 144. 146. 147. 165. 185. 204. 227. 256. 271. 349. 471. 538. 559. 560. 568. 583. 595. 697. 922. IV. 75. V. 49.

That such motions may happen has been shewn in the third article, which contains instances of the conjunction of stars with nebulosities of which they cannot have been formed, and which must, consequently, have been united by motion. We also know that nebulæ are subject to great changes in their appearance, which proves that some of the nebulous matter in their composition must be in motion; instances of which have been given in the luminous nebulosity of the constellation of Orion.* It may therefore be easily conceived that any moving patch of nebulous matter must be arrested on its meeting with stars; especially if several of them should happen to be pretty near each other; in which case there will be, as it were, a net spread out for intercepting every nebulosity that comes within the reach of their attraction.

II, 304. "Three or four stars of various sizes are mixed "with pretty strong nebulosity."

III, 165. "Five or six stars forming a parallelogram, are "mixed with very faint milky nebulosity."

III, 697. "Several small stars are contained in faint nebu-"losity about 3 or 4 minutes long and ¼ broad." See fig. 10.

IV, 75. "Three stars of about the 9th or 10th magnitude "are involved in pretty strong milky nebulosity."

This collection of thirty-seven objects, consisting of 2, 3, 4, 5, 6, or more small stars that are mixed with nebulosity, contains a variety of instances in which the effect that has been mentioned of the interception of the nebulous matter may have taken place. It is very obvious that nothing positive can be said about the formation of so many starry-nebulous patches; for unless by long continued observation of the same patches

* Phil. Trans. for 1811, p. 320.

we could be acquainted with every change that may happen in the nebulosity or in the magnitude of the stars which apparently compose them, their real union and construction must remain unknown. We can only hint, that every nebulosity which is carried into the region of a small patch of stars will probably be gradually arrested and absorbed by them, and that thus the *growth of stars* may be continued.

8. *Of objects of an ambiguous construction.*

From objects consisting decidedly of stars, but which either have nebulosity mixed with them, or are in such situations as to be seen in the same line with nebulosity, I proceed to give an account of some others, of which my observations have not ascertained into what order we ought to class them.

It has been remarked, on a former occasion, that clusters of stars, when they are at a great distance, may assume a nebulous appearance.* This may be experienced by observing a certain celestial object with a telescope of an inferior space-penetrating power, through which it will be seen as a nebula; whereas with an instrument which has a higher degree of this power, its appearance will be a mixture of nebulosity and stars; and if this power of the telescope is of a still higher order, the stars of the same object will then be distinctly perceived: the nebulosity will no longer be seen, and the object will be entitled to be placed into the rank of clusters of stars.

Other objects there are, where a greater space-penetrating power will only increase the brightness of the nebulosity, and at the same time make the tinge of it more uniformly united

* Phil. Trans. for 1811, page 270.

and of a milky appearance, which will decide it to be purely nebulous.

But when an object is of such a construction, or at such a distance from us, that the highest power of penetration, which hitherto has been applied to it, leaves it undetermined whether it belong to the class of nebulæ or of stars, it may then be called ambiguous. As there is, however, a considerable difference in the ambiguity of such objects, I have arranged 71 of them into the following four collections.*

The first contains seven objects that may be supposed to consist of stars, but where the observations hitherto made, of either their appearance or form, leave it undecided into which class they should be placed.

Connoiss. 31 is "A large nucleus with very extensive ne-"bulous branches, but the nucleus is very gradually joined to "them. The stars which are scattered over it appear to be be-"hind it, and seem to lose part of their lustre in the passage of "their light through the nebulosity; there are not more of them "scattered over the nebula than there are over the immediate "neighbourhood. I examined it in the meridian with a mirror "24 inches in diameter, and saw it in high perfection; but its "nature remains mysterious. Its light, instead of appearing "resolvable with this aperture, seemed to be more milky."

* See seventy-one ambiguous objects, in four collections.

First collection II, 400. III, 379. 693. 745. V, 2. Connoiss. 1. 31.

Second collection I, 46. 50. II, 27. 78. 79. 180. 195. 199. 207. 554. 609. 771. 822. 850. 855. III, 3. 101. 239. 399. 455. 696. 725. 743. IV, 22. Connoiss. 57. 70.

Third collection I, 44. 47. II. 47. 48. 76. 105. 202. 279. 283. 469. 473. 500. 608. 808. III, 47. 53. 55. 134. 288. 580. 747. 910. V, 1. VI, 38. Connoiss. 81. 82.

Fourth collection I, 52. 103. 122. 249. 288. II, 4. 84. 584. V, 3. VI, 15, 20. Connoiss. 100.

The objects in this collection must at present remain ambiguous.

The next contains 26 nebulous objects, of which the figure has been ascertained to be round or nearly round.

II, 101 is "A pretty large, round, extremely faint easily "resolvable nebula. I can almost see the stars in it. See fig. 11.

Connoiss. 57 is "An oval nebula with an eccentric oval dark "space in the middle; there is a strong suspicion of its con- "sisting of stars. The diameter, measured by the large 10 "feet telescope, is 1′ 28″,8."

The globular form of the objects in this collection, which is deduced from their round figure, will so far ascertain the manner of their construction, that they must either be still in a condensed state purely nebulous, or else, if consisting of stars, that they must be already in a far advanced order of compression, and only appear nebulous on account of their very great distance from us. A middle state between the progressive condensation of a globular nebula and a cluster of stars can have no existence; because a globular nebulosity when condensed can only produce a single star. There is, however, a possibility that a mass of nebulous matter in motion may be intercepted by a globular cluster, in which case the nebulosity must soon assume the form of the cluster, and will finally be absorbed by it.

In the third collection I have placed 26 nebulæ, which not only are described as easily resolvable, but in most of which some stars have actually been seen.

II, 500 is "A very large, easily resolvable, extended, nebu- "lous object. I see a few of the largest stars in it. See fig. 12.

Here the uncertainty in which the descriptions leave us, is that the objects in this collection may be either clusters of stars mixed with nebulosity, or that in consequence of the great distance and compression of the small stars composing a cluster which contains no nebulosity, it may put on the nebulous appearance.

The fourth collection contains 12 nebulous objects, of which the description makes it probable that they belong to the order of clusters of stars.

I, 949 is "A considerably bright extended nebula about 4′ "long and 2′ broad; it is easily resolvable, and I suppose with "a higher power and longer attention the stars would become "visible. It is brighter about the middle."

Connoiss. 100 is "A nebula of about 10′ in diameter, but "there is in the middle of it, a small, bright cluster of supposed "stars."

9. *Of the sidereal part of the Heavens.*

The foregoing observations have proved the intimate connection between the nebulous and sidereal condition; and although in passing from one to the other we have met with a number of ambiguous objects, it has been seen that the apparent uncertainty of their construction is only the consequence of the want of an adequate power in our telescopes, to shew them of their real form. We have indeed no reason to expect that an increase of light and distinctness of our telescopes would free us from ambiguous objects; for by improving our power of penetrating into space, and resolving those which we have at present, we should probably reach so many new objects that others, of an equally obscure construction, would

obtrude themselves, even in greater number, on account of the increased space of the more distant regions of their situation.

From stars mixed with nebulosity we are now to direct our attention to the purely sidereal part of the heavens; and as stars are the elementary parts of sidereal constructions, it will be proper to review what we know of their nature. Having already entered upon this subject in a former paper at some length,* I shall only give a few additional observations, with a summary outline of the former arguments.

The intensity of the light of a star of the first magnitude may be compared with solar light, by considering, that if the sun were removed to the distance at which we generally admit the brightest stars to be from us, its visible diameter could not exceed the 2 1 5th part of a second; and its appearance therefore would probably not differ much from the size and brightness of such stars. By reversing this argument we shall be authorised to conclude, from analogy, that stars, were they near enough, would assume the brightness, and some of them perhaps also the size, of the sun; and the consequences that have been drawn from the observations given in my paper on the nature and construction of the sun, may be legitimately applied to the stars; whence it follows that stars, although surrounded by a luminous atmosphere, may be looked upon as so many opaque, habitable, planetary globes; differing, from what we know of our own planets, only in their size, and by their intrinsically luminous appearance.

They also, like the planets, shine with differently coloured light. That of Arcturus and Aldebaran for instance, is as

* Phil. Trans. for 1785, page 68.

different from the light of Sirius and Capella, as that of Mars and Saturn is from the light of Venus and Jupiter. A still greater variety of coloured star-light has already been shewn to exist in many double stars, such as γ Andromedæ, β Cygni, and many more.* In my sweeps are also recorded the places of 9 deep garnet, 5 bright garnet, and 10 red coloured stars, of various small magnitudes from the 7th to the 12th.

By some experiments, on the light of a few of the stars of the first magnitude, made in 1798, by a prism applied to the eye-glasses of my reflectors, adjustable to any angle and to any direction, I had the following analyses.

The light of Sirius consists of red, orange, yellow, green, blue, purple, and violet.

α Orionis contains the same colours, but the red is more intense, and the orange and yellow are less copious in proportion than they are in Sirius.

Procyon contains all the colours, but proportionally more blue and purple than Sirius.

Arcturus contains more red and orange and less yellow in proportion than Sirius.

Aldebaran contains much orange, and very little yellow.

α Lyræ contains much yellow, green, blue, and purple.

The similarity of the general construction of the sun, the stars, and the planets, is also much supported by the periodical variations of the light of the stars observed in many of them;† for these variations can only be satisfactorily accounted for by admitting such stars to have a rotatory motion on their

* See Catalogue of double stars Phil. Trans. for 1782, III. 5. V. 5, &c.

† See Mr. PIGOTT's Catalogue of variable stars Phil. Trans. for 1786, page 191.

axes, like that which the sun and the planets are known to have.*

10. *Of the aggregation of Stars.*

That stars are not spread in equal portions over the celestial regions is evident to the eye of every one who directs his view to them in a clear night; but if this wanted any proof, the star-gages I have given in the Phil. Trans. for 1785, would abundantly shew that the greatest variety in their distribution takes place; for while in my sweeps many fields of view of the telescope were without a single star, others contained every assignable number, from one to more than six hundred.

In my examination of the heavens, I remarked that in many places there were patches of stars of such a particular appearance that I was induced to call them *forming* clusters. This expression was however only used to denote that some peculiar arrangement of stars in lines making different angles, directed to a certain aggregation of a few central stars, suggested the idea that they might be in a state of progressive approach to them. This tendency to clustering seemed chiefly to be visible in places that were extremely rich in stars. In order therefore to investigate the existence of a clustering power, we may expect its effects to be most visible in and near the milky way, and it is for this purpose I have distinguished the relative situation of the clusters to which I refer.†

* See Remarks on the rotatory motion of stars on their axes, Phil. Trans. for 1796, page 456.

† See twenty clusters of stars; fifteen in the milky way VII, 40. 45. VIII, 16. 18. 35. 36. 42. 47. 50. 56. 60. 61. 64. 67. Connoiss. 6; and five near it VIII, 8. 40. 41. 44. 83.

Connoiss. 6 is "A cluster of stars of various sizes contain-
" ing several lines that seem to be drawing to a centre like a
" forming cluster."

VIII, 35 is "A large cluster of stars considerably com-
" pressed and rich; some of the stars are arranged in a long
" crooked line."

VIII, 44 is "A very coarsely scattered cluster of large stars;
" they form a cross and extend over a large space; not rich."
See fig. 13. The stars about the cluster belong to the milky way.

VIII, 83 is "A cluster of scattered stars above 15′ in dia-
" meter; pretty rich and joining to the milky way, or a pro-
" jecting part of it."

The 20 objects here referred to are not given as instances of the actual formation of clusters, which, being an effect that must undoubtedly require much time, cannot be visible; but merely to draw our attention to a seemingly aggregating arrangement of the stars, which must render it probable that in regions where stars are very numerous, but unequally scattered, a clustering of them may arise from the preponderating attractions residing in different places.

11. *Of irregular Clusters.*

When clusters of stars are situated in very rich parts of the heavens, they are generally of an irregular form and very imperfectly collected; those which are in, and very near the milky way, may indeed be looked upon as so many portions of the great mass drawn together by the action of a clustering power, of which they tend to prove the existence.

I have divided the following 112 objects into two collections.

The first of them contains 80 clusters of which the size or figure has not been particularized.*

VIII, 4 is "A cluster of coarsely and irregularly scattered, "pretty large stars, of nearly one size and colour." See fig. 14.

The stars of these clusters are in general very promiscuously scattered; they are however sufficiently drawn together to shew that they form separate groups; and in many places a defalcation of the number of stars surrounding the clusters is already so far advanced as to indicate a tendency to future insulation.

The second collection contains 92 irregular clusters that are from 2 to 30′ in diameter.†

VII, 4 is "A cluster of large stars about 20 or 25′ in dia- "meter, considerably rich; it is of a coarsely circular figure."

The great number of clusters in these two collections is not only an indication that they owe their origin to a clustering power residing in the stars about their center; but the still remaining irregularity of their arrangement additionally proves that the action of the clustering power has not been exerted long enough to produce a more artificial construction. The

* See eighty irregular clusters of stars, of various unascertained sizes, fifty-three in the milky way; VII, 5. 35. 36. 42. 50. 62. 67. VIII, 4. 5. 6. 13. 15. 19. 20. 21. 22. 25. 27. 28. 30. 31. 33. 34. 37. 45. 46. 51. 52. 54. 55. 57. 58. 59. 63. 72. 76. 79. 82. 84. 85. 86. 87. Connoiss. 7. 8. 16. 18. 21. 24. 25. 26. 29. 36. 38. Eighteen near the milky way; VII, 6. 15. 46. VIII, 2. 11. 23. 43. 49. 62. 65. 68. 69. 73. Connoiss. 20. 34. 39. 41. 48: nine at a distance from it; VII, 3. 54. VIII, 7. 16. 29. 71. Connoiss. 44. 45. 73.

† See thirty-two irregular clusters from 2 to 30′ in diameter; twenty-two in the milky way; VI, 23. VII, 10. 12. 30. 52. VIII, 9. 12. 14. 17. 26. 32. 39. 48. 53. 70. 74. 77. 78. 80. 81. Connoiss. 23. 93. Ten near the milky way VI, 39. VII, 4. 11. 13. 14. 16. 32. 66. VIII, 66. 88.

length of time required for this purpose must, however, greatly depend upon the original situation of the stars exposed to the clustering power.

12. *Of Clusters variously extended and compressed.*

The outlines of clusters of stars in rich parts of the heavens, and even of those that are insulated, are seldom sufficiently defined to arrange such clusters by their figure; and as the following assortment contains some that are variously extended and differently compressed, it will be seen, from the descriptions of a few of them, that the power which has drawn the stars together must have acted under different circumstances.*

VI, 3 is "A cluster of very compressed, extremely small "stars, containing a few large ones. It is of an extended "figure, and, as it were, divided."

In this cluster the observed partial division points out the cause of its being extended, which may be ascribed to a double seat of preponderating attractions at some little distance from each other.

VI, 24 is "A very rich cluster of extremely small and very "compressed stars; it is about 6′ long and 4′ broad."

Here the stars of the cluster are not only much compressed, but the borders of it are moreover sufficiently determined to shew the limits of its extent; from which we may infer that the cluster is advancing towards insulation, and that in the end a gradual concentration may bring it to a globular form.

* See fifteen extended clusters of stars; twelve in the milky way II, 198. VI, 3. 14. 24. 36. VII, 18. 19. 27. 41. 44. 56. VIII, 3. And three near the milky way VII, 29. 64. VIII, 75.

VI, 96 is "A very compressed cluster of small, and some "large stars; extended nearly in the meridian; the most "compressed part is about 8′ long and 2′ broad, with many "stars scattered around it to a considerable distance." See fig. 15.

The construction of this cluster may have arisen from the situation of many stars in the same plane, drawn towards a centre by the clustering power, for any plane seen obliquely will have the appearance of an extended form.

VI, 64 is "A large cluster of stars, of a middling size, ir-"regularly extended, and considerably rich. The stars are "chiefly in rows."

Here each row of stars may have a different preponderating attraction, but every row will attract all the other rows; nay, from the laws of gravitation it is evident that there must be somewhere in all the rows together the seat of a preponderating clustering power, which will act upon all the stars in the neighbourhood.

13. *Of Clusters of Stars of a peculiar description.*

The great variety of ways in which the attractions of unequally scattered stars may produce a clustering power will be further exemplified in the following objects.*

VI, 55 is "A pretty compressed cluster of very small stars; "it is of an irregular figure, and has a vacancy in the middle."

This appearance may be accounted for by supposing, for instance, three, four, or a greater number of preponderating

* See six clusters of stars of a peculiar description; one in the milky way VIII, 24. Three near the milky way VII, 26. 55. Connoiss. 4; and two at a distance from it VII, 1. Connoiss. 30.

attracting centres near each other, situated so as to inclose a certain space, the stars in which, then, cannot be accumulated, while the clustering power arising from the combined attractions will be exerted on the surrounding stars.

Connoiss. 4 is "A rich cluster of considerably compressed "small stars surrounded by many straggling ones. It con- "tains a ridge of stars running through the middle from south "preceding to north following. The ridge contains 8 or 10 "pretty bright stars. All the stars are red."

The curious construction of this cluster is sufficiently accounted for by the bright stars in what is called a ridge; the small stars accumulated about it having somewhat the appearance of the shelving sides of the ridge. The observed red colour was probably owing to the low situation of the object.

VII, 26 is "A cluster of extremely small and pretty much "compressed stars, with a few large ones in the shape of a "hook."

From what has been remarked already it will not be necessary to enter into a consideration of the cause of the uncommon form of this cluster.

Connoiss. 30 is "A brilliant cluster, the stars of which are "gradually more compressed in the middle. It is insulated, "that is, none of the stars in the neighbourhood are likely to "be connected with it. Its diameter is from 2′ 40″ to 3′ 30″. "Its figure is irregularly round. The stars about the centre "are so much compressed as to appear to run together. To- "wards the north, are two rows of bright stars 4 or 5 in a "line."

In this accumulation of stars, we plainly see the exertion of a central clustering power, which may reside in a central

mass, or, what is more probable in the compound energy of the stars about the centre. The lines of the bright stars, although by a drawing made at the time of observation, one of them seems to pass through the cluster, are probably not connected with it.

14. *Of differently compressed Clusters of Stars.*

I have hitherto only considered the arrangement of stars in clusters with a view to point out that they are drawn together by a clustering power, in the same manner as the nebulous matter has, in my former paper, been proved to be condensed by the gravitating principle; but in the 41 clusters of the following two collections we shall see that it is one and the same power uniformly exerted which first condenses nebulous matter into stars, and afterwards draws them together into clusters, and which by a continuance of its action gradually increases the compression of the stars that form the clusters.

The first collection contains 33 clusters, the stars of which are considerably compressed.*

VII, 12 is "A beautiful cluster of pretty compressed stars, "near half a degree in diameter. It is considerably rich, and "most of the stars are of the same size."

The moderate compression of the stars in the clusters of this order renders them fine objects for good telescopes.

The second collection contains 8 clusters in which the

* See thirty clusters of considerably compressed stars; seventeen in the milky way VI, 16. 29. 33. 34. VII, 2. 7. 9. 22. 23. 33. 43. 65. VIII, 38. Connoiss. 11. 35. 50. 103. Fifteen near the milky way VI, 6. 22. 42. VII, 12. 17. 20. 21. 24. 34. 47. 57. 58. 59. 63. VIII, 1; and one at a distance from it Connoiss. 67.

compression of the stars is carried to a much higher degree.*

VI, 30 is "A very beautiful rich cluster of very compressed "small stars."

The clusters in this collection are also fine objects; but, on account of their higher compression, require superior telescopes.

15. *Of the gradual concentration and insulation of Clusters of Stars.*

The existence of a clustering power is nowhere so visibly pointed out as in the 39 clusters which are given in the following collection.† My remarks upon them will come with more clearness when applied to a particular description of some of them.

VI, 5 is "A beautiful cluster of very compressed small stars "of several sizes. It is of an irregular round form, about 12 "or 15′ in diameter, and the stars are gradually most com- "pressed in the middle."

Here the gradually increasing compression of the stars points out the central situation of the clustering power; the form is also that of a solid, not much differing from a globular figure; and by the outline of the cluster we may consider it as already in an advanced state of insulation; from these cir-

* See eight clusters of very compressed stars; five in the milky way VI, 27. 30. VII, 8. Connoiss. 22. 46. Two near the milky way VI, 10. VII. 48; and one at a distance from it VI, 4.

† See thirty-nine clusters of gradual concentration; twenty-one in the milky way VI, 5. 13. 17. 18. 25. 26. 28. 32. VII, 25. 28. 31. 37. 38. 39. 51. 60. 61. Connoiss. 28. 37. 52. 71. Seven near the milky way VI, 2. 21. 31. 37. 40. VII, 49. 53; and eleven at a distance from it I, 41. IV, 63. VI, 1. 8. 9. 19. Connoiss. 33. 55. 68. 74. 77.

cumstances we may therefore conclude that this cluster has been long under the influence of the clustering power. See fig. 16.

Connoiss. 68 is "A beautiful cluster of stars, extremely rich, "and so compressed that most of the stars are blended toge- "ther; it is near 3′ broad and about 4′ long, but chiefly round, "and there are very few scattered stars about."

This oval cluster is also approaching to the globular form, and the central compression is carried to a high degree. The insulation is likewise so far advanced that it admits of an accurate description of the contour.

The clusters of this class are beautiful, but can hardly be seen to any advantage without a 20 feet telescope.

16. *Of globular Clusters of Stars.*

The objects of this collection are of a sufficient brightness to be seen with any good common telescope, in which they appear like telescopic comets, or bright nebulæ, and under this disguise, we owe their discovery to many eminent astronomers; but in order to ascertain their most beautiful and artificial construction, the application of high powers, not only of penetrating into space but also of magnifying are absolutely necessary; and as they are generally but little known and are undoubtedly the most interesting objects in the heavens, I shall describe several of them, by selecting from a series of observations of 34 years some that were made with each of my instruments, that it may be a direction for those who wish to view them to know what they may expect to see with such telescopes as happen to be in their possession.*

* See fourteen globular clusters of stars. One in the milky way Connoiss. 19.

Oct. 4, 1810. 40 feet telescope. Space-penetrating power 191,68. Magnifying power 280. "Having been a sufficient "time at the telescope to prepare the eye properly for seeing "minute objects, the 72d of the *Connoissance des temps* came "into the field. It is a very bright object."

"It is a cluster of stars of a round figure, but the very faint "stars on the outside of globular clusters are generally a "little dispersed so as to deviate from a perfect circular form. "The telescopes which have the greatest light shew this best."

"It is very gradually extremely condensed in the centre, "but with much attention, even there, the stars may be dis- "tinguished."

"There are many stars in the field of view with it, but they "are of several magnitudes totally different from the exces- "sively small ones which compose the cluster."

"It is not possible to form an idea of the number of stars "that may be in such a cluster; but I think we cannot esti- "mate them by hundreds."

"The diameter of the cluster is about $\frac{1}{3}$ of the field, which gives "1′ 59″,6." See fig. 17.

Sept. 4, 1799. 40 feet telescope, power 240. "I examined "the 2d of the Connoiss. It appeared very brilliant and lu- "minous."

"The scattered stars were brought to a good, well deter- "mined focus, from which it appears that the central con- "densed light is owing to a multitude of stars that appeared "at various distances behind and near each other. I could "actually see and distinguish the stars even in the central

Four near the milky way Connoiss. 10. 12. 56. 80; and nine at a distance from it Connoiss. 2. 3. 5. 13. 15. 53. 72. 79. 92.

"mass. The Rev. Mr. VINCE, Plumian Professor of Astronomy "at Cambridge, saw it in the same telescope as described."

May 27, 1791. 40 feet telescope, power 370. "The 5th "of the Connoiss. is a beautiful cluster of stars; I counted "about 200 of them; but the middle of it is so compressed "that it is impossible to distinguish the stars."*

January 5, 1807. 20 feet telescope. Space-penetrating power 75,08. Magnifying power 157.8. "The 56th of the "Connoiss. is a globular cluster of very compressed and "very small stars. They are gradually more compressed "towards the centre."

May 26, 1786. 20 feet telescope. "The 80th of the Con- "noiss. is a beautiful, round cluster of extremely minute and "very compressed stars about 9 or 4' in diameter; by the "increasing compression of the stars the cluster is very gra- "dually much brighter in the middle."

May 16, 1787. 20 feet telescope. "The 13th of the Con- "noiss. is a most beautiful cluster of stars. It is exceedingly

* A 40 feet telescope should only be used for examining objects that other instruments will not reach. To look through one larger than required is loss of time, which, in a fine night, an astronomer has not to spare; but it ought to be known that the opportunities of using the 40 feet reflector are rendered very scarce by two material circumstances. The first is the changeable temperature of the atmosphere, by which the mirror is often covered with the condensation of vapour upon its surface, which renders it useless for many hours; and in cold weather by freezing upon it for the whole night, and even for weeks together; for the ice cannot be safely taken off till a general thaw removes it. The next is that, with all imaginable care, the polish of a mirror exposed like that in the 40 feet telescope, though well covered up, will only preserve its required lustre and delicacy about two years. The three observations I have given must consequently be looked upon as having been made by three different mirrors; but if we will have superior views of the heavens, we must submit to circumstances that cannot easily be altered.

" compressed in the middle and very rich. The most com-
" pressed part of it is round and is about 2 or $2\frac{1}{2}'$ in diameter,
" the scattered stars which belong to it extend to 8 or 9′ in
" diameter, but are irregular."*

Sept. 24, 1810. Large 10 feet Newtonian telescope. Space-penetrating power 75,89. Magnifying powers 71. 108. 171. 220. " The 3d of the Connoiss. is one of the globular clus-
" ters; very brilliant and beautiful. The compression of the
" stars begins to increase pretty suddenly from the outside at
" $\frac{1}{4}$ of the radius, and continues gradually up to the centre, its
" diameter taking in the outside is full half of the field of the
" glass magnifying 171 times, which gives 4′ 30″."

Nov. 23, 1805. Large 10 feet. " The 15th of the Con-
" noiss. is perfectly round, and insulated. The accumulation
" of the stars towards the centre is more sudden than the 19th
" of the Connoiss. and the scattered stars extend proportion-
" ally much farther. Its diameter is $\frac{1}{6}$ of the field of the glass
" which magnifies 108 times, that is to say 4′ 0″. It passes
" the wire in 13″,0 of time which by calculation gives only
" 2′ 11″,3, but I rely more on the estimation by the known
" field of view which is 24′ 0″; because the limits of the clus-
" ter cannot be properly fixed upon for a transit."

Jan. 13, 1806. Large 10 feet. " The 79th of the Connoiss.
" is a cluster of stars of a globular construction, and certainly

* The 10 feet telescope, on account of the moderate weight of the mirror and the proportionally long wooden tube, has the great advantage that with proper precaution it may be used in any temperature. Sometimes, however, a sudden change from cold to heat towards morning has put a stop to the observations of the night. The mirror will also preserve an excellent polish for several years; and having a second one ready to supply the place of that which is in use the instrument may always be ready for observation.

" extremely rich. Towards the centre the stars are extremely " compressed, and even a good way from it. With 171 the " diameter is a little less than $\frac{1}{4}$ of the field, and with 220 a " little more; the field of one being 9′ 0″, and of the other " 8′ 0″, a mean of both gives the diameter of the cluster 2′ 50″, " but I suppose that the lowness of the situation prevents my " seeing the thinly scattered stars, so that this cluster is pro- " bably larger than it appears."*

Common 10 feet telescope. Space-penetrating power 28,67. " When the 19th of the Connoiss. is viewed with a magnify- " ing power of 120, the stars are visible; the cluster is insu- " lated; some of the small stars scattered in the neighbourhood " are near it; but they are larger than those belonging to the " cluster. With 240 it is better resolved, and is much con- " densed in the center. With 300 no nucleus or central body " can be seen. The diameter with the 10 feet is 3′ 16″, and " the stars in the centre are too accumulated to be separately " seen."

7 feet telescope, space-penetrating power 20,25. " The " 53d of the Connoiss. with 118 is easily resolvable, and some " of the stars may be seen."

It will not be necessary to add that the two last mentioned globular clusters, viewed with more powerful instruments, are of equal beauty with the rest; and from what has been said it

* The large 10 feet telescope is in a considerable degree subject to the obstructions arising from change of temperature, and tarnish; but as it can be directed to any part of the heavens in a few minutes, and is easily prepared for observation, it becomes a very useful instrument when the clearness of the atmosphere is interrupted by flying clouds; or when the place of an object not visible in the finder, or night-glass, is to be ascertained.

is obvious that here the exertion of a clustering power has brought the accumulation and artificial construction of these wonderful celestial objects to the highest degree of mysterious perfection.

17. *Of more distant globular Clusters of Stars.*

The objects contained in this assortment are so like those of the foregoing collection that in my observations I have called them miniatures of the former. Small instruments cannot reach them, I shall therefore describe them as they appear when proper powers are applied to them.*

VI, 35 is "A cluster of very faint exceedingly compressed "stars, about one minute in diameter. It is the next step to "an easily resolvable nebula."

VI, 11 is "A cluster of stars about 1½ or 2 minutes in dia- "meter. It is a good miniature of the 19th of the Connoiss. "not only with respect to the size of the cluster, but also with "regard to the mutual distance and the reduced magnitude "of the stars of which it consists."

Connoiss. 9 is "A cluster of very compressed and extremely "small stars. It is a miniature of the 53d."

Connoiss. 14 is "Like an extremely bright, easily resolv- "able round nebula; but with a power of 300 I can see the "stars of it. It resembles the 10th of the Connoiss. which "probably would put on the same appearance as this, were "it removed half its distance farther from us. The stars are "much condensed in the middle."

* See eleven miniature globular clusters of stars, five in the milky way VI, 11. 12. 35. Connoiss. 9. 62. One near the milky way Connoiss. 14; and five at a distance from it I, 78. III, 709. VI, 7. 41. Connoiss. 75.

Connoiss. 62 is " Extremely bright, round, very gradually " brighter in the middle, easily resolvable, about 4′ in diameter. " With 240 and strong attention I see the stars of it. It is a " miniature of the 3d of the Connoiss."

I, 78 is " Very bright, suddenly much brighter in the " middle, round, about 3′ in diameter. I take it to be a clus- " ter of stars, as it seems to be a miniature of the 2d of the " Connoiss."

III, 709 is " Very faint, round, very gradually brighter in " the middle; about $2\frac{1}{2}$ minutes in diameter." A later observation says " I can perceive some of the stars."

Connoiss. 75 is " A globular cluster of stars, and is a minia- " ture of the third."

I have supposed the clusters of this class to be at a greater distance from us than those of the preceding collection, because the stars of which they are composed are more minute than those of the clusters of which I have called them miniatures; their compression is also closer, and the size of the whole is much contracted, all which particulars are readily explained by admitting them to be more distant. This argument, however, does not extend so far as to exclude a real difference which there may be in different clusters, not only in the size but also in the number and arrangement of the stars.

18. *Of still more distant globular Clusters of Stars.*

It has already been shewn in the 8th article, that when our telescopes have extended vision as far as they can reach with distinctness, they will still shew objects at a greater distance if they are sufficiently bright to be seen, although we should

not be able to ascertain exactly into what class we ought to place them; but as it frequently happened that I saw three objects in succession, the first of which was a brilliant globular cluster of stars, the second a miniature of the former of which the stars could but just be perceived, and the third in every respect a similar miniature of the second as the second was of the first, but in which the stars, though suspected, were no longer to be distinguished, I called them second miniature globular clusters. The following collection contains five of them.*

I, 45 is "A bright round nebula, much brighter in the middle, "but the brightness decreasing very gradually. It is a perfect "miniature of VI, 12, which is itself a miniature cluster of the "19th of the Connoiss."

I, 48 is "A miniature of the 9th of the Connoiss." (which is itself a miniature of the 53d) "I suppose if I had looked "long enough, I might have perceived some of the stars which "compose it."

I, 147 is "A miniature of the 62d of the Connoiss. which is "a miniature of the 3d."

I, 51 and Connoiss. 69 are second miniatures of the 53d.

19. *Of a recurrence of the ambiguous limit of observation.*

In the 16th article I have given a description of the most magnificently constructed sidereal systems; and very little doubt can be entertained but that the objects of the 17th and 18th articles are of the same nature, and are only less beautiful

* See five second miniature globular clusters of stars in the milky way I, 45. 48. 51. 147. Connoiss. 69.

in their appearance as they are gradually more remote. It has already been shewn in the 8th article, that in passing from faint nebulosity to the suspected sidereal condition, we cannot avoid meeting with ambiguous objects, to which I must now add, that the same critical situation will again occur, when from the distinctly sidereal appearance we endeavour to penetrate gradually farther into space. In consequence of this remark, it seems probable that among the numerous globular nebulæ which have been given in my last paper, many beautiful clusters of stars may lie concealed. To this we may add, that several of the great number of objects which have been given as stellar nebulæ, and are probably at a still greater distance from us, may be the last glimpses we can have of such clusters of stars as the 77th of the Connoissance des Temps, which will nearly put on the stellar appearance when it is viewed in a very good common telescope.

This ambiguity, however, being the necessary consequence of the faintness or distance of objects, when seen through telescopes that are not sufficiently powerful to shew them as they are, will not affect any of the arguments that have been used to establish the existence of a clustering power, the effects of which have gradually been traced from the first indication of clustering stars, through irregular as well as through more artificially arranged clusters, up to the beautiful globular form.

The extended views I have taken, in this and my former papers, of the various parts that enter into the construction of the heavens, have prepared the way for a final investigation of the universal arrangement of all these celestial bodies in

space; but as I am still engaged in a series of observations for ascertaining a scale whereby the extent of the universe, as far as it is possible for us to penetrate into space, may be fathomed, I shall conclude this paper by pointing out some inferences which the continuation of the action of the clustering power enables us to draw from the observations that have been given.

20. *Of the breaking up of the milky way.*

The milky way is generally represented in astronomical maps as an irregular zone of brightness encircling the heavens, and my star gages have proved its whitish tinge to arise from accumulated stars, too faint to be distinguished by the eye. The great difficulty of giving a true picture of it is a sufficient excuse for those who have traced it on a globe, or through the different constellations of an Atlas Cœlestis, as if it were a uniform succession of brightness. It is, however, evident that, if ever it consisted of equally scattered stars, it does so no longer; for, by looking at it in a fine night, we may see its course between the constellations of Sagittarius and Perseus affected by not less than eighteen different shades of glimmering light, resembling the telescopic appearances of large easily resolvable nebulæ; but in addition to these general divisions, the observations detailed in the preceding pages of this paper, authorise us to anticipate the breaking up of the milky way, in all its minute parts, as the unavoidable consequence of the clustering power arising out of those preponderating attractions which have been shewn to be every where existing in its compass.

One hundred and fifty-seven instances have been given of clusters situated within the extent of the milky way, and their places are referred to in nine preceding articles. They may also be found in Bode's Atlas Cœlestis, whose delineation of this bright zone I have taken for a standard. To these must be added 68 more, which are in the less rich parts, or what may be called the vanishing borders of the milky way: for this immense stratum of stars does not break off abruptly, as generally represented in maps, but gradually becomes invisible to the eye when the stars are no longer sufficiently numerous to cause the impression of milkiness.

Now, since the stars of the milky way are permanently exposed to the action of a power whereby they are irresistibly drawn into groups, we may be certain that from mere clustering stars they will be gradually compressed through successive stages of accumulation, more or less resembling the state of some of the 268 objects by which, in the tenth and six succeeding articles, the operation of the clustering power has been laid open to our view, till they come up to what may be called the ripening period of the globular form, and total insulation; from which it is evident that the milky way must be finally broken up, and cease to be a stratum of scattered stars.

We may also draw a very important additional conclusion from the gradual dissolution of the milky way; for the state into which the incessant action of the clustering power has brought it at present, is a kind of chronometer that may be used to measure the time of its past and future existence; and although we do not know the rate of going of this mysterious

chronometer, it is nevertheless certain, that since the breaking up of the parts of the milky way affords a proof that it cannot last for ever, it equally bears witness that its past duration cannot be admitted to be infinite.

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Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.

Fig. 7.

Fig. 8.

Fig. 9.

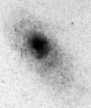


Fig. 10.

Fig. 11.

Fig. 12.

Fig. 13.

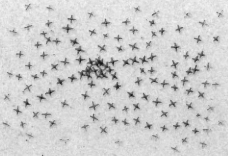


Fig. 14.

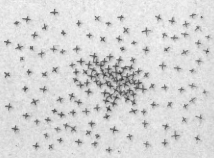


Fig. 15.

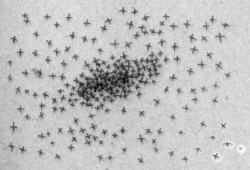


Fig. 16.

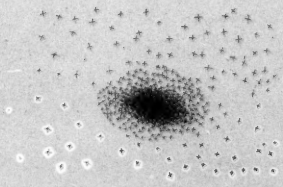
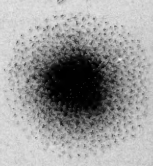


Fig. 17.



METEOROLOGICAL JOURNAL,

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ROYAL SOCIETY,

BY ORDER OF THE

PRESIDENT AND COUNCIL.

MDCCCXIV.

METEOROLOGICAL JOURNAL

for January, 1813.

1813	Time. H.	Time. M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer. °	Winds. Points.	Winds. Str.	Weather.
Jan. 1	8	0	42	48	29,82	75	W	1	Cloudy.
	4	0	45	49	29,89	75	W	1	Cloudy.
2	9	0	41	48	30,12	76	S	1	Fine.
	4	0	45	50	30,14	75	W	1	Cloudy.
3	9	0	37	49	30,40	77	W	1	Thick and cloudy.
	4	0	42	47	30,30	75	W	1	Cloudy.
4	8	0	46	47	30,37	75	S	1	Cloudy.
	4	0	42	50	30,28	72	SE	1	Cloudy.
5	9	0	37	47	30,08	70	SW	1	Cloudy. Wind in the night.
	4	0	42	50	30,03	72	S	1	Cloudy.
6	8	0	42	48	29,88	76	SW	2	Cloudy.
	4	0	47	51	29,74	76	SW	2	Cloudy.
7	8	0	43	50	29,77	79	W	1	Thick fog.
	4	0	44	53	29,73	77	SW	1	Cloudy.
8	8	0	44	50	29,29	77	W	1,2	Cloudy.
	4	0	44	51	29,39	67	W	1,2	Cloudy.
9	8	0	33	49	29,65	72	W	1	Hazy.
	4	0	40	52	29,79	73	W	1	Cloudy.
10	8	0	33	49	29,92	68	N	1	Fine.
	4	0	37	50	29,92	65	NNE	1	Fine.
11	8	0	34	46	29,71	71	SE	1	Cloudy.
	4	0	38	49	29,68	67	E	1	Cloudy.
12	8	0	32	47	29,81	69	SE	1	Fine.
	4	0	30	49	29,78	66	SE	1	Cloudy.
13	8	0	34	46	29,62	72	SE	1	Cloudy.
	4	0	37	48	29,53	73	SE	1	Rain.
14	8	0	36	45	29,59	75	SE	1	Cloudy.
	4	0	38	49	29,65	75	E	1	Cloudy.
15	8	0	35	48	29,76	75	W	1	Cloudy.
	4	0	38	49	29,88	73	N	1	Cloudy.
16	8	0	34	47	30,05	73	SSE	1	Fine.
	4	0	41	49	30,12	71	S	1	Fine.

Rain this Month 0,336 Inches.

METEOROLOGICAL JOURNAL

for January, 1813.

1813	Time. H.	Time. M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer. °	Winds. Points.	Winds. Str.	Weather.
Jan. 17	8	0	34	46	30,20	74	SE	1	Cloudy.
	4	0	37	46	30,23	72	E	1	Cloudy.
18	8	0	32	45	30,17	73	SE	1	Cloudy.
	4	0	34	46	30,09	72	SE	1	Cloudy. [night.
19	8	0	31	44	30,03	73	SE	1	Cloudy. Some snow in the
	4	0	34	47	30,16	73	N	1	Cloudy.
20	8	0	33	44	30,32	72	SE	1	Cloudy.
	4	0	35	47	30,31	71	E	1	Cloudy.
21	8	0	32	44	30,34	70	W	1	Cloudy.
	4	0	35	47	30,39	70	NE	1	Cloudy.
22	8	0	32	45	30,46	70	E	1	Cloudy.
	4	0	32	49	30,49	68	E	1	Fine.
23	8	0	27	45	30,36	71	E	1	Thick and foggy.
	4	0	33	47	30,26	72	W	1	Cloudy.
24	8	0	30	44	30,56	75	E	1	Cloudy. Snow in the night.
	4	0	32	43	30,38	69	E	1	Fine.
25	8	0	29	42	30,44	73	NE	1	Cloudy.
	4	0	34	45	30,45	70	E	1	Cloudy.
26	8	0	34	44	30,45	69	NE	1	Cloudy.
	4	0	37	46	30,44	67	N	1	Cloudy.
27	8	0	34	44	30,46	68	NNE	1	Cloudy.
	4	0	36	47	30,48	70	N	1	Cloudy.
28	8	0	28	42	30,45	72	S	1	Foggy.
	4	0	34	47	30,40	70	NW	1	Fine.
29	8	0	25	44	30,37	72	W	1	Thick and cloudy.
	4	0	34	47	30,34	72	NE	1	Cloudy.
30	8	0	31	45	30,48	75	NE	1	Cloudy.
	4	0	34	46	30,49	69	NE	1	Cloudy.
31	8	0	35	44	30,43	70	NW	1	Cloudy.
	4	0	42	45	30,44	68	N	1	Cloudy.

Rain this Month 0,136 Inches.

METEOROLOGICAL JOURNAL

for February, 1813.

1813	Time. H.	M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer. °		Winds. Points.	Str.	Weather.
Feb. 1	8	0	34	42	30,45	70		W	1	Thick and cloudy.
	4	0	40	45	30,31	70		N	1	Cloudy.
2	8	0	33	43	30,31	74		E	1	Cloudy.
	4	0	37	47	30,32	71		N	1	Cloudy.
3	8	0	35	45	30,36	72		W	1	Cloudy.
	4	0	42	48	30,39	65		N	1	Cloudy.
4	8	0	38	46	30,45	72		W	1	Cloudy.
	4	0	41	49	30,40	65		NW	1	Cloudy.
5	8	0	37	48	30,22	68		S	1	Cloudy.
	4	0	40	53	30,08	66		S	1,2	Fine. [night.
6	8	0	40	48	29,73	73		SW	2	Rain. Blowed hard in the
	4	0	44	49	29,83	73			1	
7	8	0	43	49	29,93	73		S	1	Fine.
	4	0	41	48	29,97	73			1	Fine.
8	8	0	45	49	29,74	76		SW	2	Cloudy.
	4	0	48	52	29,67	73		W	2	Rain.
9	8	0	44	51	29,63	72		W	1	Fine. [thunder at 2 PM.
	4	0	44	54	29,68	67		W	1	Fine. A storm of hail and
10	8	0	37	50	29,89	71		W	1	Fine.
	4	0	44	53	29,98	62		W	1	Fine.
11	8	0	37	50	30,05	71		W	1	Fine.
	4	0	44	56	29,98	66		S	1	Fine.
12	8	0	44	52	29,69	73		S	2	Rain.
	4	0	49	54	29,61	71		S	2	Rain.
13	8	0	43	52	29,17	72		W	1	Cloudy.
	4	0	45	56	29,27	70		W	1	Cloudy.
14	8	0	45	52	29,27	75		S	2	Rain.
	4	0	49	53	29,18	76		S	2	Rain.
15	8	0	45	51	29,32	73		S	1,2	Fine.
	4	0	49	55	29,33	68		W	2	Cloudy.
16	8	0	43	52	29,26	70		W	1,2	Fine.
	4	0	46	57	29,41	61		SSW	2	Fine.

Rain this Month 1,794 Inches.

METEOROLOGICAL JOURNAL
for February, 1813.

1813	Time. H.	Time. M.	Therm. without.	Therm. within.	Barom.	Hygrometer.		Winds. Points.	Winds. Str.	Weather.
			°	°	Inches.					
Feb. 17	8	0	42	53	29.39	70		SW	1	Cloudy.
	4	0	47	55	29.28	70		SW	1	Rain.
18	8	0	46	53	29.34	63		W	2	Fine.
	4	0	48	56	29.60	58		NW	2	Rain.
19	8	0	45	54	29.91	74		SSW	1	Cloudy.
	4	0	50	57	29.74	74		SW	1	Cloudy.
20	8	0	41	54	29.66	71		W	1	Cloudy.
	4	0	48	57	29.95	64		SW	1,2	Fine.
21	8	0	51	54	29.81	74		W	1	Cloudy.
	4	0	53	55	29.75	70		SW	1,2	Cloudy.
22	8	0	48	54	29.68	67		SW	2	Fine.
	4	0	51	56	29.67	67		S	2	Rain.
23	8	0	43	55	29.83	71		W	1	Rain.
	4	0	45	57	29.95	63		NW	1	Fine.
24	8	0	38	52	30,07	70		W	1	Fine.
	4	0	46	56	30,10	65		W	1	Fine.
25	8	0	37	51	30,23	71		W	1	Cloudy.
	4	0	46	54	30,06	70		SW	1,2	Cloudy.
26	8	0	47	54	29.94	71		SW	2	Cloudy.
	4	0	49	55	29,81	70		SW	2	Rain.
27	8	0	39	53	29,96	65		W	1	Fine.
	4	0	46	55	30,14	58		NW	2	Cloudy.
28	8	0	40	52	30,04	69		W	1	Hazy.
	4	0	49	55	30,44	60		N	1,2	Fine.

Rain this Month 1,794 Inches.

METEOROLOGICAL JOURNAL

for March, 1813.

1813	Time. H.	M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer. °	Winds. Points.	Str.	Weather.
Mar. 1	8	0	39	51	30,38	70	SW	1,2	Fine.
	4	0	47	55	30,29	65	SW	2	Cloudy.
2	8	0	48	53	30,19	73	W	1	Cloudy.
	4	0	48	57	30,22	66	N	1	Cloudy.
3	8	0	42	53	30,11	70	SW	1	Cloudy.
	4	0	48	55	30,07	66	N	1	Cloudy.
4	8	0	37	52	30,39	68	W	1	Cloudy.
	4	0	50	55	30,38	61	W	1	Cloudy.
5	8	0	46	54	30,33	75	W	1	Thick and cloudy.
	3	0	52	57	30,39	66	W	1	Cloudy.
6	8	0	40	54	30,41	68	W	1	Fine.
	3	0	50	58	30,39	60	NW	1	Fine.
7	8	0	41	53	30,49	68	W	1	Fine.
	3	0	49	53	30,42	61	W	1	Cloudy.
8	7	0	47	53	30,28	67	W	1	Fine, rather hazy.
	3	0	52	58	30,26	63	NW	1	Cloudy.
9	7	0	47	54	30,22	70	W	1	Thick and cloudy.
	3	0	50	57	30,11	65	WNW	1	Cloudy.
10	7	0	40	53	29,95	70	E	1	Rain.
	3	0	42	54	29,92	66	NNW	1	Cloudy
11	7	0	32	52	30,01	67	NE	1,2	Fine.
	3	0	37	57	30,06	62	N by W	1,2	Fine.
12	7	0	30	50	30,24	65	N	1,2	Fine.
	3	0	36	55	30,29	61	E	1	Cloudy.
13	7	0	30	49	30,34	68	N	1	Hazy.
	3	0	39	52	30,28	60	NW	1	Cloudy.
14	7	0	35	53	30,23	65	S	1	Cloudy.
	3	0	40	48	30,18	69	SW	1	Cloudy.
15	7	0	44	47	30,11	76	SW	1	Cloudy.
	3	0	49	51	30,16	73	S	1	Cloudy.
16	7	0	45	49	30,22	77	S	1	Cloudy.
	3	0	49	53	30,15	76	SE	1	Cloudy. Very dark at 2 PM

Rain this Month 0,558 Inches.

METEOROLOGICAL JOURNAL
for March, 1813.

1813	Time. H.	M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer. °		Winds. Points.	Str.	Weather.
Mar. 17	7	0	38	50	30,07	74		NNE	1	Fine.
	3	0	50	58	29,96	59		N	1	Fine.
18	7	0	38	50	29,93	71		NW	1	Cloudy.
	3	0	55	58	29,92	63		NE	1	Fine.
19	7	0	45	54	29,94	71		W	1	Cloudy.
	3	0	51	58	29,87	61		E	1	Fine.
20	7	0	47	55	29,75	71		S	2	Cloudy.
	3	0	49	55	29,73	65		S	1	Cloudy.
21	7	0	42	53	29,96	69		S	1	Fair.
	3	0	49	53	29,93	62		S	2	Cloudy.
22	7	0	49	53	29,76	74		S	2	Cloudy.
	3	0	53	57	29,80	60		W	1	Fine.
23	7	0	38	52	30,30	67		W	1	Cloudy.
	3	0	48	63	30,34	55		N	1	Fine.
24	7	0	40	52	30,32	64		W	1	Cloudy and hazy.
	3	0	45	60	30,29	69		NW	1	Cloudy.
25	7	0	45	53	30,06	76		W	1	Rain.
	3	0	46	57	30,22	70		N	1	Cloudy.
26	7	0	42	54	30,44	68		N	1	Cloudy.
	3	0	48	59	30,46	59		N	1	Fine.
27	7	0	38	49	30,59	69		SW	1	Fine.
	3	0	53	58	30,53	60		N	1	Cloudy.
28	7	0	45	55	30,51	72		W	1	Fine.
	3	0	59	57	30,43	63		W	1	Cloudy.
29	7	0	53	56	30,38	72		W	1	Cloudy.
	3	0	58	58	30,28	65		SW	1	Cloudy.
30	7	0	51	56	30,15	69		W	1	Hazy.
	3	0	51	59	30,11	69		NE	1	Cloudy.
31	7	0	45	56	29,96	68		SE	1	Cloudy and foggy.
	3	0	52		29,82	56		N	1	Fine.

Rain this Month 0,558 Inches.

METEOROLOGICAL JOURNAL

for April, 1813.

1813	Time. H.	Time. M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer. °		Winds. Points.	Winds. Str.	Weather.
Apr. 1	7	0	43	56	29,31	68		SSW	1	Rain.
	3	0	45	58	29,26	68		SW	1	Cloudy.
2	7	0	38	55	29,33	68		WSW	1	Fine.
	3	0	44	60	29,45	63		W	1	Fine.
3	7	0	36	54	29,56	66		SW	1	Cloudy.
	3	0	43	57	29,64	58		W	1	Fine.
4	7	0	37	53	29,83	66		W	1	Fine.
	3	0	44	56	29,86	58		N	1	Cloudy.
5	7	0	39	50	29,92	65		W	1	Rain.
	3	0	47	49	29,82	70		W	1	Rain.
6	7	0	47	52	29,85	75		SW	1	Cloudy.
	3	0	55	56	29,92	67		NNE	1	Cloudy.
7	7	0	47	54	30,02	71		NW	1	Cloudy.
	3	0	55	57	29,99	63		SSW	1	Cloudy.
8	7	0	49	55	29,98	70		E	0	Thick and cloudy.
	3	0	57	62	29,95	62		SE	1	Fine.
9	7	0	47	58	30,04	70		SW	0,1	Hazy.
	3	0	59	65	30,06	56		SW	0,1	Fair.
10	7	0	51	58	30,14	69		NE	0,1	Fine.
	3	0	63	60	30,16	63		E	1	Fine.
11	7	0	48	58	30,19	70		E	1	Cloudy.
	3	0	56	62	30,16	61		E	1	Fair.
12	7	0	43	57	30,17	68		E	1	Hazy and cloudy.
	3	0	63	63	30,16	62		SE	1	Fine.
13	7	0	50	59	30,32	68		NE	1	Fine.
	3	0	60	65	30,36	57		NE	1	Fair.
14	7	0	47	57	30,42	67		SE	1	Cloudy.
	3	0	57	61	30,32	60		E	1	Fair.
15	7	0	48	57	30,23	66		SW	1	Hazy.
	3	0	54	63	30,11	62		NW	1	Fine.
16	7	0	54	59	30,13	69		N	1	Cloudy.
	3	0	61	62	30,00	57		NW	1	Fine.

Rain this Month 1,114 Inches.

METEOROLOGICAL JOURNAL

for April, 1813.

1813	Time. H.	M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer. °		Winds. Points.	Str.	Weather.
Apr. 17	7	0	52	58	29,87	70		W	1	Cloudy.
	3	0	53	62	29,80	61		WNW	1	Fine.
18	7	0	44	58	30,16	60		NE	1	Fair.
	3	0	53	60	30,21	50		NW	1	Fair.
19	7	0	53	57	30,14	65		NW	1	Fine.
	3	0	60	60	30,13	61		SE	1	Cloudy.
20	7	0	53	58	30,17	63		NW	1	Hazy.
	3	0	60	62	30,13	55		NW	1	Cloudy.
21	7	0	49	58	30,12	64		N by E	1	Fine.
	3	0	55	60	30,07	57		NW	1	Cloudy.
22	7	0	42	56	30,14	61		N	1	Fine.
	3	0	46	57	30,07	65		N	1	Cloudy.
23	7	0	40	55	30,14	65		NE	1	Cloudy.
	3	0	43	57	30,16	57		E	1	Cloudy.
24	7	0	40	54	30,16	59		E	1	Cloudy.
	3	0	40	52	30,09	63		E	1	Cloudy.
25	7	0	41	51	30,03	68		N	1	Rain.
	3	0	49	52	29,93	69		E	1	Rain.
26	7	0	46	51	29,76	76		E	1	Cloudy.
	3	0	52	56	29,78	60		S	1	Fine.
27	7	0	45	52	29,66	74		NE	1	Rain.
	3	0	48	52	29,46	75		N	1	Cloudy.
28	7	0	45	52	29,43	76		NE	1	Rain.
	3	0	47	53	29,46	72		N	1	Cloudy.
29	7	0	43	52	29,63	74		N	1	Cloudy.
	3	0	45	52	29,65	65		N	2	Cloudy.
30	7	0	40	51	29,65	75		E	1	Rain.
	3	0	47	52	29,67	75		E	1	Cloudy.

Rain this Month 1,114 Inches.

METEOROLOGICAL JOURNAL

for May, 1813.

1813	Time. H.	Time. M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer. °		Winds. Points.	Winds. Str.	Weather.
May 1	7	0	45	52	29.68	77		E	1	Rain.
	3	0	49	52	29.68	75		E	1	Cloudy
2	7	0	47	52	29.74	76		E	1	Cloudy.
	3	0	52	52	29.81	71		N	1	Cloudy.
3	7	0	49	52	29.84	76		SE	1	Cloudy.
	3	0	59	56	29.82	61		E	1	Fine.
4	7	0	54	54	29.80	75		E	1	Rain.
	3	0	61	56	29.81	70		W	1	Cloudy.
5	7	0	54	55	29.98	76		NE	1	Cloudy.
	3	0	61	58	29.98	63		N	1	Fine.
6	7	0	52	56	29.99	74		NE	1	Cloudy.
	3	0	61	60	29.84	66		E	1	Cloudy.
7	7	0	55	58	29.78	80		S	1	Cloudy.
	3	0	60	61	29.77	60		W	1	Fine.
8	7	0	52	57	29.76	72		E	1	Cloudy.
	3	0	63	59	29.68	61		E	1	Cloudy.
9	7	0	56	58	29.65	67		E	1	Cloudy.
	3	0	61	62	29.68	59		E	1	Fine.
10	7	0	56	59	29.83	70		W	1	Cloudy.
	3	0	63	61	29.86	60		W	1	Cloudy.
11	7	0	55	59	29.99	67		W	1	Cloudy.
	3	0	62	60	29.96	60		S	1	Rain.
12	7	0	57	59	29.67	70		W	1	Rain.
	3	0	62	62	29.68	60		W	2	Fine.
13	7	0	56	60	29.73	64		W	1	Fine.
	3	0	60	62	29.64	62		S	1	Cloudy.
14	7	0	54	60	29.56	67		S	1	Cloudy.
	3	0	60	61	29.49	59		S	1	Cloudy.
15	7	0	54	60	29.54	65		SW	1,2	Cloudy.
	3	0	60	61	29.61	63		S	1	Cloudy.
16	7	0	50	59	29.65	66		S	2	Rain.
	3	0	58	61	29.52	59		W	2	Cloudy.

Rain this Month 2.630 Inches.

METEOROLOGICAL JOURNAL

for May, 1818.

1813	Time. H.	M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer. °	Winds. Points.	Str.	Weather.
May 17	7	0	52	58	29,82	65	NW	2	Cloudy.
	3	0	59	60	29,89	58	NW	1	Cloudy.
18	7	0	50	58	29,75	70	N	1	Rain.
	3	0	56	59	29,83	63	NE	1	Cloudy.
19	7	0	53	58	29,90	67	S	1	Cloudy.
	3	0	59	61	29,76	65	W	1	Cloudy.
20	7	0	57	58	29,59	67	W	1	Rain.
	3	0	59	59	29,47	63	W	1	Rain.
21	7	0	49	57	29,53	61	E	2	Fine.
	3	0	45	58	29,49	66	SW	2	Rain.
22	7	0	52	56	29,62	62	SW	1	Fine.
	3	0	55	57	29,65	60	E	1	Fine.
23	7	0	49	56	29,69	66	W	1	Fine.
	3	0	53	57	29,60	65	SW	1	Rain.
24	7	0	51	56	29,49	66	W	1	Fine.
	3	0	58	58	29,54	56	NW	1	Fine.
25	7	0	51	56	29,69	60	NW	1	Fine.
	3	0	56	57	29,70	64	SW	1	Cloudy.
26	7	0	54	56	29,60	66	W	1	Rain.
	3	0	53	58	29,79	62	NW	1	Hail, thunder, and lightning.
27	7	0	50	55	30,09	63	W	1	Fine.
	3	0	58	57	30,14	57	SW	1	Fine.
28	7	0	52	56	30,15	65	E	1	Fine.
	3	0	61	60	30,04	55	SE	1	Fine.
29	7	0	55	58	29,92	68	E	1	Fine.
	3	0	62	60	29,99	68	E	1	Cloudy.
30	7	0	62	59	30,07	65	E	1	Cloudy.
	3	0	62	60	30,05	63	E	1	Cloudy.
31	7	0	63	60	30,07	65	W	1	Cloudy.
	3	0	66	64	30,06	59	S	1	Fine.

Rain this Month 2,030 Inches.

METEOROLOGICAL JOURNAL

for June, 1819.

1813	Time. H.	Time. M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer. °		Winds. Points.	Winds. Str.	Weather.
June 1	7	0	62	62	30,11	57		NW	1	Fine.
	3	0	69	66	30,07	54		SE	0,1	Fine.
2	7	0	64	63	29,99	65		NE	1	Cloudy.
	3	0	70	69	29,97	60		E	1	Fine.
3	7	0	61	65	30,06	62		W	1	Fine.
	3	0	70	67	30,14	62		N	1	Cloudy.
4	7	0	57	64	30,29	61		NW	1	Cloudy.
	3	0	63	64	30,22	59		N	1	Cloudy.
5	7	0	56	59	30,10	60		N	1	Fine.
	3	0	59	62	29,96	62		N	1	Cloudy.
6	7	0	53	61	29,84	64		N	1	Hazy.
	3	0	55	59	29,76	60		NE	1	Cloudy.
7	7	0	53	57	29,78	64		N	1	Cloudy.
	3	0	62	62	29,81	55		E	1	Fine.
8	7	0	56	60	29,82	63		E	1	Fine.
	3	0	56	64	29,75	63		E	1	Fine.
9	7	0	58	60	29,59	67		E	1	Cloudy.
	3	0	63	62	29,53	63		SW	1	Fine.
10	7	0	58	61	29 61	66		W	1	Cloudy.
	3	0	68	65	29,71	52		W	1	Fine.
11	7	0	58	62	29,93	62		SW	1	Fair.
	3	0	65	64	29,93	54		SSW	1	Fine.
12	7	0	61	62	29,83	63		S	1	Cloudy.
	3	0	60	63	30,00	58		W	1	Fine.
13	7	0	58	62	30,10	61		W	1	Fine.
	3	0	66	63	30,12	55		W	1	Cloudy.
14	7	0	58	62	30,08	64		W	1	Cloudy.
	3	0	56	62	29,92	63		SW	1	Rain.
15	7	0	56	62	29,73	65		W	1	Cloudy.
	3	0	56	65	29,73	65		W	1	Fine.
16	7	0	56	62	30,00	62		N	1	Cloudy.
	3	0	62	65	30,01	53		W	1	Fine

Rain this Month 1,351 Inches.

METEOROLOGICAL JOURNAL
for June, 1819.

1813	Time. H.	Time. M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer. °		Winds. Points.	Winds. Str.	Weather.
June 17	7	0	54	61	30,05	61		NW	1	Fine.
	3	0	58	61	30,06	57		N	1	Rain.
18	7	0	51	60	30,11	66		N	1	Cloudy.
	3	0	59	62	30,09	55		NE	1	Fine.
19	7	0	51	59	30,17	64		N	1	Cloudy.
	3	0	54	60	30,15	61		N	1	Cloudy.
20	7	0	52	59	30,07	61		N	1	Fine.
	3	0	60	59	30,07	55		N by W	1	Cloudy.
21	7	0	53	58	30,27	63		N	1	Fine.
	3	0	60	60	30,26	54		N	1	Fine.
22	7	0	55	59	30,23	63		NNE	1	Cloudy.
	3	0	61	61	30,22	57		N	2	Cloudy.
23	7	0	52	59	30,28	60		N	1	Fine.
	3	0	62	62	30,23	54		N	1	Fair.
24	7	0	58	60	30,13	64		N	1	Cloudy.
	3	0	63	61	30,21	61		NE	1	Cloudy.
25	7	0	57	60	30,30	65		E	1	Cloudy.
	3	0	64	63	30,26	58		BSE	1	Fine.
26	7	0	55	61	30,28	67		E	1	Fine.
	3	0	65	64	30,19	58		S	1	Fine.
27	7	0	58	61	30,19	65		N	1	Fine.
	3	0	65	65	30,11	56		E	1	Cloudy.
28	7	0	54	61	30,08	65		NW	1	Cloudy.
	3	0	63	62	29,98	63		SSE	1	Rain.
29	7	0	57	62	29,88	67		SW	1	Fine.
	3	0	63	63	29,79	62		SSE	1	Cloudy. Thunder at 1 PM.
30	7	0	51	61	29,72	67		W	1	Rain.
	3	0	56	61	29,66	68		S	2	Rain.

Rain this Month 1,351 Inches.

METEOROLOGICAL JOURNAL

for July, 1813.

1813	Time. H.	M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer. °	Winds. Points.	Str.	Weather.
July 1	7	0	57	61	29,73	69	W	1	Cloudy.
	3	0	62	62	29,78	64	W	1	Cloudy.
2	7	0	58	61	29,83	63	W	1	Fine.
	3	0	57	62	29,87	57	NW	1	Cloudy.
3	7	0	53	60	29,97	67	NNE	1	Cloudy.
	3	0	60	61	30,08	60	N	1	Cloudy.
4	7	0	52	59	30,23	62	N	1	Fine.
	3	0	63	61	30,24	56	N	1	Cloudy.
5	7	0	55	60	30,29	65	N	1	Fine.
	3	0	63	62	30,26	54	NW	1	Fine.
6	7	0	60	61	30,16	62	S	1	Fine.
	3	0	62	62	30,07	59	S	1	Fine.
7	7	0	62	62	29,85	59	S	1	Fair.
	3	0	68	66	29,70	54	S	1	Cloudy.
8	7	0	60	63	29,75	63	SW	1	Cloudy.
	3	0	65	64	29,73	59	S	1	Cloudy.
9	7	0	58	63	29,66	65	W	1	Fine.
	3	0	68	67	29,66	53	W	1	Fine.
10	7	0	62	64	29,90	64	E	1	Cloudy and hazy.
	3	0	69	67	29,96	53	W	1	Fine.
11	7	0	59	64	30,02	64	NE	1	Fine.
	3	0	67	65	30,01	59	N	1	Cloudy.
12	7	0	60	64	29,99	65	W	1	Fair.
	3	0	69	67	29,93	54	W by N	1	Fair.
13	7	0	58	65	29,87	67	W	1	Cloudy.
	3	0	66	66	29,83	60	SW	1	Cloudy.
14	7	0	60	64	29,74	66	W	1	Cloudy.
	3	0	66	65	29,71	61	W	1	Cloudy.
15	7	0	59	64	29,68	68	W	1	Rain.
	3	0	61	65	29,70	67	N	1	Rain.
16	7	0	55	64	29,82	68	W	1	Fine.
	3	0	65	66	29,86	56	W	1	Fine.

Rain this Month 2,092 Inches.

METEOROLOGICAL JOURNAL

for July, 1813.

1813	Time. H.	M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer. °	Winds. Points.	Str.	Weather.
July 17	7	0	59	59	29,98	66	W	1	Fine.
	3	0	66	66	29,94	58	SSW	1	Cloudy.
18	7	0	55	64	29,98	67	W	1	Fair.
	3	0	68	66	29,97	54	SW	1	Cloudy.
19	7	0	61	63	29,85	62	SE	1	Cloudy.
	3	0	66	65	29,76	60	ESE	1	Cloudy.
20	7	0	62	64	29,64	69	E	1	Cloudy.
	3	0	68	68	29,61	55	E	1	Fine.
21	7	0	60	65	29,61	67	NE	1	Cloudy.
	3	0	63	65	29,67	64	N	1	Cloudy.
22	7	0	59	65	29,67	69	SW	1	Cloudy.
	3	0	66	66	29,64	60	S	1	Cloudy.
23	7	0	59	64	29,60	66	W	1	Cloudy.
	3	0	66	66	29,58	67	W	1	Cloudy.
24	7	0	50	64	29,58	67	W	1	Cloudy.
	3	0	64	62	29,62	65	S	1	Rain.
25	7	0	63	65	29,54	71	W	1	Cloudy.
	3	0	68	67	29,54	59	W	1	Fine.
26	7	0	59	64	29,66	64	W	1	Fine.
	3	0	63	65	29,67	67	W	1	Fine.
27	7	0	57	64	29,90	68	W	1	Cloudy.
	3	0	67	66	29,97	60	W	1,2	Cloudy.
28	7	0	58	64	30,19	68	N	1	Cloudy.
	3	0	68	67	30,21	57	S	1	Fine.
29	7	0	58	65	30,28	65	W	1	Fair.
	3	0	69	70	30,24	56	E	1	Fair.
30	7	0	63	66	30,09	68	E	1	Hazy.
	3	0	73	73	29,97	55	S	1	Fair.
31	7	0	61	69	30,03	63	NW	1	Fair.
	3	0	71	70	30,05	56	W	1	Cloudy.

Rain this Month 2,092 Inches.

METEOROLOGICAL JOURNAL

for August, 1813.

1813	Time. H.	Time. M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer. °	Winds. Points.	Winds. Str.	Weather.
Aug. 1	7	0	62	67	30,10	61	W	1	Fine.
	3	0	69	68	30,11	56	W	1	Cloudy.
2	7	0	60	67	30,06	65	W	1	Cloudy.
	3	0	69	68	30,01	57	W	1	Cloudy.
3	7	0	63	67	29,93	67	W	1	Cloudy.
	3	0	68	69	29,87	58	SW	1	Fine.
4	7	0	59	66	29,92	65	W	1,2	Cloudy.
	3	0	67	68	29,89	54	NW	1	Cloudy.
5	7	0	59	66	29,74	65	SE	1	Cloudy.
	3	0	66	67	29,55	60	SW	2	Cloudy.
6	7	0	61	65	29,67	65	W	1	Cloudy.
	3	0	66	67	29,82	58	W	2	Cloudy.
7	7	0	57	65	30,01	65	N	1	Fine.
	3	0	68	70	30,02	53	W	1	Fine.
8	7	0	61	66	30,03	64	W	1	Cloudy.
	3	0	68	67	30,01	58	N	1	Cloudy.
9	7	0	58	66	30,03	65	N	1	Fine.
	3	0	65	69	30,11	57	N	1	Cloudy.
10	7	0	58	66	30,22	65	W	1	Fine.
	3	0	67	67	30,22	58	WNW	1	Cloudy.
11	7	0	62	66	30,21	62	SW	1	Cloudy.
	3	0	69	68	30,17	57	SSW	1	Cloudy.
12	7	0	62	66	30,12	62	W	1	Fair.
	3	0	70	72	30,04	55	W	1	Fine.
13	7	0	62	67	30,11	65	WNW	1	Hazy.
	3	0	65	70	30,17	57	N	1,2	Cloudy.
14	7	0	57	66	30,24	63	W	1	Fine.
	3	0	67	67	30,11	55	W	1	Cloudy.
15	7	0	63	66	29,96	65	NW	1	Hazy.
	3	0	68	69	29,95	54	NW	1	Fine.
16	7	0	62	66	29,92	59	NW	1	Fine.
	3	0	65	66	29,92	54	NW	1	Cloudy.

Rain this Month 0,565 Inches.

METEOROLOGICAL JOURNAL
for August, 1813.

1813	Time. H.	M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer. °		Winds. Points.	Str.	Weather.
Aug. 17	7	0	59	65	29,95	65		W	1	Cloudy.
	3	0	67	66	29,92	57		N	1	Cloudy.
18	7	0	58	65	29,95	68		W	1	Hazy.
	3	0	67	68	29,95	57		W	1	Cloudy.
19	7	0	57	65	30,16	62		W	1	Fine.
	3	0	64	68	30,20	53		N	1	Fine.
20	7	0	56	64	30,29	61		W	0,1	Fine.
	3	0	64	67	30,23	51		NE	1	Fine.
21	7	0	50	63	30,26	62		W	1	Fine.
	3	0	57	62	30,11	57		W	1	Fine.
22	7	0	56	62	29,97	61		W	1	Cloudy.
	3	0	62	63	29,86	60		NW	1	Cloudy.
23	7	0	51	61	30,04	64		W	1	Fine.
	3	0	55	65	30,16	62		NE	2	Fine.
24	7	0	51	62	30,32	64		NE	1	Fair.
	3	0	63	64	30,32	55		E	1	Cloudy.
25	7	0	55	61	30,38	62		NE	0	Hazy.
	3	0	63	64	30,34	56		NE	1	Cloudy.
26	7	0	57	62	30,33	63		NE	1	Fine.
	3	0	63	65	30,29	56		E	1	Cloudy.
27	7	0	56	62	30,27	65		N	1	Cloudy.
	3	0	61	64	30,26	65		N	1	Cloudy.
28	7	0	56	62	30,20	64		NE	1	Cloudy.
	3	0	60	62	30,20	60		E	1	Cloudy.
29	7	0	57	61	30,22	72		NE	1	Cloudy.
	3	0	62	63	30,24	63		N	1	Cloudy.
30	7	0	54	61	30,33	65		NNE	1	Cloudy.
	3	0	63	63	30,29	56		N	1	Cloudy.
31	7	0	57	61	30,31	67		E	1	Cloudy.
	3	0	63	64	30,21	60		E	1	Cloudy.

Rain this Month 0,765 Inches.

METEOROLOGICAL JOURNAL

for September, 1813.

1813	Time. H.	Time. M.	Therm. without. o	Therm. within. o	Barom. Inches.	Hygrometer.	Winds. Points.	Winds. Str.	Weather.
Sept. 1	7	0	56	61	30,03	69	N	1	Cloudy.
	3	0	63	63	29,88	61	E	1	Cloudy.
2	7	0	58	62	29,77	70	S	1	Fine.
	3	0	62	64	29,84	62	W	1	Fine.
3	7	0	56	61	29,93	72	SSE	1	Fine.
	3	0	64	67	29,89	61	S	1	Cloudy.
4	7	0	61	63	29,77	69	S	2,3	Cloudy.
	3	0	63	64	29,71	68	SSW	1	Cloudy.
5	7	0	58	62	29,66	73	SW	0	Rain.
	3	0	62	69	29,65	58	SW	1	Fine.
6	7	0	56	62	29,12	72	E	2	Rain.
	3	0	58	70	29,35	58	W	2	Fine.
7	7	0	53	61	29,44	63	W	1	Fine.
	3	0	57	63	29,53	61	NNW	2	Rain.
8	7	0	47	59	29,65	66	W	1	Fine.
	3	0	52	60	29,67	63	NW	1	Cloudy.
9	7	0	45	58	29,82	68	W	1	Hazy.
	3	0	57	74	29,99	56	N	1	Fine.
10	7	0	47	58	30,23	66	W	1	Fine.
	3	0	58	67	30,30	57	W	1	Fine.
11	7	0	54	59	30,26	67	W	1	Fine.
	3	0	62	66	30,21	60	W	1	Fine.
12	7	0	58	60	30,11	70	W	1	Fine.
	3	0	63	66	30,00	59	SW	1	Fine.
13	7	0	53	60	30,03	65	W	1	Fair.
	3	0	61	64	30.05	56	W	1,2	Cloudy.
14	7	0	52	60	30,02	67	W	1	Fair.
	3	0	60	64	30,07	56	NW	1	Fine.
15	7	0	47	59	30,19	67	W	1	Fine.
	3	0	61	63	30.18	59	SW	1	Cloudy.
16	7	0	58	61	30,17	69	W	1	Cloudy.
	3	0	61	64	30.21	65	NW	1	Fine.

Rain this Month 0,861 Inches.

METEOROLOGICAL JOURNAL

for September, 1813.

1813	Time. H.	M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer.		Winds. Points.	Str.	Weather.
Sept. 17	7	0	56	61	30,30	70		N	1	Hazy.
	3	0	63	64	30,32	60		S	1	Cloudy.
18	7	0	57	62	30,28	68		E	1	Cloudy.
	3	0	63	63	30,17	62		E	1	Cloudy.
19	7	0	54	61	30,08	67		E	1	Fair.
	3	0	61	63	30,03	60		E	1	Fine.
20	7	0	53	60	29,99	69		E	1	Fine.
	3	0	60	66	29,97	60		E	1	Fine.
21	7	0	52	61	29,97	70		N	1	Cloudy.
	3	0	63	62	29,96	60		N	1	Cloudy.
22	7	0	54	61	29,99	72		N	1	Cloudy.
	3	0	61	63	30,01	63		N	1	Fine.
23	7	0	52	61	30,05	73		NE	1	Cloudy.
	3	0	60	62	30,00	64		NE	1	Cloudy.
24	7	0	55	60	30,08	70		N	1	Cloudy.
	3	0	60	63	30,16	61		E	1	Fine.
25	7	0	52	59	30,25	69		N	1	Fine.
	3	0	58	60	30,20	62		E	1	Cloudy.
26	7	0	54	60	30,11	71		N	1	Cloudy.
	3	0	60	62	30,14	60		E	1	Fine.
27	7	0	52	58	30,10	65		N	1	Fine.
	3	0	60	61	30,03	61		E	1	Fair.
28	7	0	53	59	30,01	73		N	1	Hazy.
	3	0	57	60	30,03	61		N	1	Cloudy.
29	7	0	50	57	30,15	65		N	1	Cloudy.
	3	0	57	57	30,18	56		NE	1	Fine.
30	7	0	52	61	30,23	65		N	1	Cloudy.
	3	0	58	50	30,16	57		E	1	Fine.

Rain this Month 0,861 Inches.

METEOROLOGICAL JOURNAL

for October, 1813.

1813	Time. H.	M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer. °		Winds. Points.	Str.	Weather.
Oct. 1	8	0	47	56	29,91	68		N	1	Cloudy.
	3	0	60	60	29.79	61		E	1	Cloudy.
2	8	0	49	55	29.75	67		N	1	Cloudy.
	3	0	55	57	29.74	64		E	1	Cloudy.
3	8	0	51	56	29.78	65		N	1	Cloudy.
	3	0	57	59	29,32	62		E	1	Fair.
4	8	0	50	56	29,88	70		E	1	Rain.
	3	0	57	58	29,73	77		SE	1	Rain.
5	8	0	58	59	29,72	73		W	1	Cloudy.
	3	0	62	61	29,81	60		W	1	Fair.
6	8	0	57	59	29,87	73		SE	1	Cloudy.
	3	0	61	60	29,87	65		E	1	Cloudy. [der in the night.
7	8	0	60	61	29,57	81		S	1	Cloudy. Much rain and thun-
	3	0	59	61	29,49	81		W	1	Rain.
8	8	0	53	59	29,82	73		W	1	Fine.
	3	0	58	60	29,60	75		W	1	Cloudy.
9	8	0	56	59	29,48	71		W	1	Cloudy.
	3	0	60	63	29,52	59		NW	1	Fine.
10	8	0	54	58	29,51	70		SW	1	Rain.
	3	0	58	61	29,44	64		SW	1	Cloudy. [the night.
11	8	0	51	57	28,98	68		NW	2	Rain. Much wind and rain in
	3	0	56	58	29,34	61		W	1	Cloudy.
12	8	0	46	56	29.79	72		W	1	Fine.
	3	0	55	57	29,80	65		S	1	Cloudy. [the night.
13	8	0	55	57	29.42	73		W	2,3	Rain. Much wind and rain in
	3	0	51	58	29,61	60		N	2	Cloudy.
14	8	0	40	53	29.54	70		NNW	1	Fine.
	3	0	48	55	29,69	61		W	1	Fine.
15	8	0	45	52	29,61	67		SW	1	Cloudy.
	3	0	49	53	29,39	70		SW	1	Rain.
16	8	0	43	51	29.24	70		W	1	Fine.
	3	0	48	53	28,97	70		SE	1	Rain.

Rain this Month 3,387 Inches.

METEOROLOGICAL JOURNAL

for October, 1819.

1813	Time. H.	Time. M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer. °		Winds. Points.	Winds. Str.	Weather.
Oct. 17	8	0	50	52	28,67	69		W	2	Cloudy.
	3	0	50	53	28,74	70		N	1	Rain.
18	8	0	39	50	29.34	70		W	0,1	Fine.
	3	0	49	53	29,50	62		W	1	Cloudy.
19	8	0	36	44	29,67	70		E	1	Fine.
	3	0	49	52	29,73	63		E	1	Fair.
20	8	0	41	49	29.70	72		E	1	Cloudy.
	3	0	50	53	29,59	70		E	1,2	Cloudy.
21	8	0	47	51	29,49	75		E	1	Rain.
	3	0	52	54	29,66	70		S	1	Fine.
22	8	0	50	53	29,79	75		W	1	Cloudy.
	3	0	54	57	29,79	71		E	1	Fine.
23	8	0	51	54	29,91	74		NE	1	Cloudy.
	3	0	53	56	29,94	69		E	1	Cloudy.
24	8	0	51	55	29,92	73		E	1	Cloudy.
	3	0	52	54	29,86	71		E	1	Cloudy.
25	8	0	48	53	29,85	72		N	1	Cloudy.
	3	0	50	57	29,90	65		E	1	Fine.
26	8	0	41	51	30,13	68		NE	1	Cloudy.
	3	0	45	53	30,15	61		N	1	Fair.
27	8	0	41	49	29,90	63		N	1	Cloudy.
	3	0	42	53	29,85	69		NE	1	Cloudy.
28	8	0	38	49	29,81	71		N	1	Fine.
	3	0	43	53	29,85	72		NE	1,2	Rain.
29	8	0	37	48	29,94	71		NE	1	Fine.
	3	0	44	52	29,94	69		N	1	Fine.
30	8	0	33	47	29,75	70		W	1	Foggy.
	3	0	46	51	29,50	68		S	2	Cloudy.
31	8	0	37	48	29,03	64		W	1	Fine.
	3	0	49	52	29,03	64		W	1	Fine.

Rain this Month 3,187 Inches.

METEOROLOGICAL JOURNAL
for November, 1813.

1813	Time. H.	Time. M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer. °	Winds. Points.	Winds. Str.	Weather.
Nov. 1	8	0	38	48	29,29	74	WNW	1	Fine.
	3	0	46	49	29,44	62	NW	1	Fine.
2	8	0	37	48	29,74	71	W	1	Fine.
	3	0	45	52	29,60	70	NW	1	Rain.
3	8	0	44	49	29,86	69	NNE	1	Fine.
	3	0	47	53	30,09	65	N	1	Fine.
4	8	0	35	48	30,35	69	W	1	Fine.
	3	0	43	54	30,35	65	W	1,2	Hazy.
5	8	0	37	50	30,34	69	E	0	Thick fog.
	3	0	46	55	30,30	66	S	1	Cloudy.
6	8	0	38	50	30,07	70	E	1	Hazy.
	3	0	44	53	29,92	64	S	1	Fine.
7	8	0	42	49	29,73	70	E	0	Cloudy.
	3	0	44	50	29,76	73	S	1	Cloudy.
8	8	0	47	50	29,40	73	SW	1	Cloudy.
	3	0	49	55	29,41	66	S	2	Rain, thunder, and lightning.
9	8	0	44	51	29,47	69	W	1	Fine.
	3	0	49	56	29,61	62	SW	2	Rain.
10	8	0	46	52	29,71	70	SSE	1	Cloudy.
	3	0	49	56	29,54	69		1	Cloudy.
11	8	0	46	53	29,68	69	W	1	Hazy.
	3	0	50	58	29,75	64	W	1	Fine.
12	8	0	52	56	29,60	69	W	2	Cloudy.
	3	0	49	58	29,53	72	W	1	Rain.
13	8	0	37	52	29,54	70	W	1	Fine.
	3	0	45	55	29,54	66	N	1	Fine.
14	8	0	34	50	29,61	71	W	0	Hazy.
	3	0	43	53	29,62	69	W	0	Fine.
15	8	0	41	50	29,20	68	W	1	Cloudy.
	3	0	46	54	29,24	64	W	1	Cloudy.
16	8	0	36	49	29,43	67	W	1	Fine.
	3	0	43	52	29,42	64	W	1,2	Cloudy.

Rain this Month 0,947 Inches.

METEOROLOGICAL JOURNAL

for November, 1813.

1813	Time. H.	Time. M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer. °	Winds. Points.	Winds. Str.	Weather.
Nov. 17	8	0	38	49	29,07	68	NW	2	Rain. Wind and rain in the night.
	3	0	37	52	29,21	70	NW	1	Cloudy.
18	8	0	35	48	29,49	70	W	1	Fine.
	3	0	42	52	29,64	65	W	1	Fine.
19	8	0	39	49	29,72	70	S	1	Rain.
	3	0	48	53	29,66	75	NW	1	Rain.
20	8	0	48	53	29,77	75	W	1	Fine.
	3	0	53	56	29,86	70	SW	1	Fine.
21	8	0	49	53	29,92	72	S	1	Cloudy.
	3	0	51	54	29,93	66	SSE	1	Cloudy.
22	8	0	48	53	29,93	74	SE	1	Cloudy.
	3	0	52	56	29,95	70	S	1	Fine.
23	8	0	41	53	29,99	72	SE	1	Foggy.
	3	0	46	56	29,98	74		1	Cloudy.
24	8	0	44	53	29,94	75	N	1	Foggy.
	3	0	40	52	30,15	74	NNE	1	Hazy.
25	8	0	37	52	30,15	68	NE	1	Cloudy.
	3	0	42	54	30,12	68	E	1	Cloudy.
26	8	0	39	50	30,12	70	N	1	Cloudy.
	3	0	40	53	30,09	64	E	1	Cloudy.
27	8	0	37	50	30,01	68	N	1	Thick and hazy.
	3	0	39	51	29,95	67	N	1	Cloudy.
28	8	0	35	48	29,92	66	N	0	Cloudy.
	3	0	39	50	29,92	65	NE	1	Cloudy.
29	8	0	37	47	29,96	62	E	1	Cloudy.
	3	0	37	49	29,99	62	E	1	Fine.
30	8	0	31	46	29,88	65	E	1	Cloudy.
	3	0	32	48	29,71	63	E	1	Cloudy.

Rain this Month 0,947 Inches.

METEOROLOGICAL JOURNAL

for December, 1813.

1813	Time. H.	M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer. °	Winds. Points.	Str.	Weather.
Dec. 1	8	0	31	42	29,48	66	E	1	Fine.
	3	0	34	47	29.54	64	E	1	Cloudy.
2	8	0	34	44	29,49	66	E	1	Cloudy.
	3	0	37	47	29,30	73	E	1	Rain.
3	8	0	38	45	29,09	75	E	1	Rain.
	3	0	39	47	29,15	73	NE	1	Cloudy.
4	8	0	39	47	29,33	75	N	1	Cloudy.
	3	0	41	50	29,35	74	NNE	1	Rain.
5	8	0	39	47	29,48	75	N	0,1	Cloudy.
	3	0	42	47	29,57	73	N	1	Cloudy.
6	8	0	37	46	29,77	75	N	1	Cloudy.
	3	0	44	50	29,80	74	N	1	Cloudy.
7	8	0	37	47	29,76	74	N	1	Fine, rather hazy.
	3	0	42	49	29,78	72	E	1	Cloudy.
8	8	0	41	48	29,79	76	NNE	1	Rain.
	3	0	43	50	29,81	76	N	1	Rain.
9	8	0	40	48	29,89	76	N	1	Rain.
	3	0	43	51	29,89	74	NE	1	Rain.
10	8	0	38	48	29,97	73	NE	1	Cloudy.
	3	0	41	50	30,09	66	NNE	1,2	Cloudy.
11	8	0	37	47	30,18	67	N	1	Cloudy.
	4	0	40	50	30,21	66	E	1	Cloudy.
12	8	0	35	47	30,10	68	SE	1	Cloudy.
	3	0	36	46	30,00	68	E	1	Fine.
13	8	0	30	38	29,83	68	W	1	Cloudy.
	3	0	35	47	29,86	66	E	1	Cloudy.
14	8	0	28	43	29,95	69	W	1	Cloudy and foggy.
	3	0	33	46	29,95	70	W	1	Cloudy and foggy.
15	9	0	27	41	29,90	71	W	0,1	Hazy thick weather.
	3	0	34	46	29,86	71	W	1	Fine.
16	8	0	35	43	29,56	74	S	1,2	Cloudy.
	3	0	40	48	29,46	78	S	1	Cloudy.

Rain this Month 0,328 Inches.

METEOROLOGICAL JOURNAL

for December, 1819.

1813	Time. H.	Time. M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer. °	Winds. Points.	Winds. Str.	Weather.
Dec. 17	8	0	46	48	29.40	78	S	1	Cloudy.
	3	0	48	50	29.22	79	S	1	Rain.
18	8	0	48	50	29.16	80	S	1	Rain.
	3	0	50	53	29.17	75	SW	1	Rain.
19	8	0	45	50	29.19	76	SW	1	Cloudy.
	3	0	47	53	29.19	75	SW	1	Fine.
20	8	0	39	48	29.31	76	W	1	Foggy.
	3	0	42	52	29.43	73	W	1	Fine, rather hazy.
21	8	0	33	47	29.62	75		0	Thick fog.
	3	0	43	51	29.55	75	S	1	Cloudy.
22	8	0	42	49	29.53	74	SW	1	Fine.
	3	0	44	52	29.54	74	SW	1	Cloudy.
23	9	0	37	49	29.78	74	W	1	Thick cloudy weather.
	3	0	42	52	29.92	74	SW	1	Cloudy.
24	8	0	45	51	29.94	77	SW	1	Rain.
	3	0	48	51	29.99	77	SW	1	Rain.
25	9	0	47	52	30.06	77	SW	1	Rain.
	3	0	49	51	30.07	77	W	1	Rain.
26	8	0	42	51	30.28	73	N	1	Hazy.
	3	0	45	51	30.40	68	W	1	Fine.
27	8	0	30	47	30.53	73	E	1	Foggy.
	3	0	31	49	30.50	72	E	1	Foggy.
28	8	0	29	45	30.39	75	E		Thick fog.
	3	0	32	47	30.36	75	E		Thick fog.
29	8	0	27	43	30.35	76	E	1	Thick fog.
	3	0	30	47	30.35	75	E	1	Thick fog.
30	8	0	26	42	30.36	75	E	1	Thick fog.
	3	0	35	45	29.33	70	E	1	Thick fog.
31	8	0	28	41	30.31	76	E	1	Thick fog.
	3	0	35	46	30.30	75	E	1	Fine, but rather hazy.

Rain this Month 0,918 Inches.

1813.	Thermometer without.			Thermometer within.			Barometer.*			Hygrometer.			Rain.†
	Greatest height.	Least height.	Mean height.	Greatest height.	Least height.	Mean height.	Greatest height.	Least height.	Mean height.	Greatest height.	Least height.	Mean height.	
	Deg.	Deg.	Deg.	Deg.	Deg.	Deg.	Inches.	Inches.	Inches.	Deg.	Deg.	Deg.	Inches
January	47	25	36,2	53	42	47,0	30,49	29,29	30,10	79	65	72,0	0,336
February	53	33	43,6	57	42	52,1	30,45	29,17	29,84	76	58	69,2	1,794
March	59	30	45,2	63	47	53,4	30,59	29,73	30,18	77	55	67,2	0,558
April	63	36	46,6	65	49	54,6	30,42	29,26	29,94	76	50	65,3	1,104
May	66	45	55,8	64	52	58,0	30,15	29,47	29,77	80	55	65,3	2,630
June	70	51	58,9	69	57	61,8	30,30	29,53	30,01	68	52	61,0	1,351
July	73	52	61,9	73	59	64,5	30,29	29,54	29,87	71	53	62,5	2,092
August	70	50	61,4	72	61	65,6	30,38	29,55	30,09	72	51	60,1	0,565
September	64	45	56,9	74	57	61,8	30,32	29,12	29,99	73	56	64,4	0,861
October	62	33	49,6	63	44	54,5	30,15	28,67	29,63	81	59	68,7	3,387
November	53	31	42,5	58	46	51,8	30,35	29,07	29,77	75	62	68,4	0,947
December	50	26	38,3	53	38	47,7	30,53	29,09	29,79	80	64	73,1	0,328
Whole year			49,7			56,1			29,91			66,4	15,953

* The quicksilver in the bason of the barometer, is 81 feet above the level of low water spring tides at Somerset-house.

† The Society's Rain Gage is 114 feet above the same level, and 75 feet 6 inches above the surrounding ground.

By another Rain Gage placed at a few feet distant from the former and 11 feet 6 inches lower, the quantity of rain this year appears to have been 18,296 inches.

Variation of the Magnetic Needle, September, 1813, 24° 16′ 40″ West.